A SEARCH FOR NEUTRINO OSCILLATIONS
IN A MUON NEUTRINO BEAM

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A SEARCH FOR NEUTRINO OSCILLATIONS
IN A MUON NEUTRINO BEAM

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This is a report on a search for $\nu_\mu \rightarrow \nu_e$ oscillations using the E776 detector at Brookhaven National Laboratory. The detector consisted of a large fiducial mass electromagnetic calorimeter followed by a toroidal muon spectrometer and was located 1 km from the target at the AGS. $1.43 \times 10^{19}$ protons on target were collected in the wide band beam with a peak neutrino energy of 1.1 GeV. No evidence for neutrino oscillations was observed. The 90% confidence limits obtained are $\Delta m^2 \leq 0.107 \, eV^2$ for large mixing angle, and $\sin^2 2\theta \leq 7.7 \times 10^{-3}$ in the limit of large $\Delta m^2$. 
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Chapter 1

Introduction

1.1 History

In 1933, Pauli was the first to suggest the existence of the neutrino when he proposed apparent violations of energy and momentum conservation observed in $\beta$-decay could be explained by an unobserved particle of very small mass which carried off the missing energy and momentum [1]. The next year, Fermi named this unobserved particle the neutrino in his treatment of $\beta$-decay [2].

In 1934, Bethe and Peierls calculated a cross section of $10^{-44}$ cm$^2$ for the reaction $\nu + (A, Z) \rightarrow e^+ + (A, Z + 1)$ [3] leading to speculation that neutrino interactions might never be observed. However, in 1953, Reines and Cowan did observe the interaction of free neutrinos using a nuclear reactor as a source [4]. The neutrons in the reactor undergo $\beta$-decay, producing electrons and anti-neutrinos. At the time, it was not clear if neutrinos or anti-neutrinos were produced in the reactor. Reines and Cowan observed the interaction $\nu p \rightarrow n e^+$ in a target of cadmium chloride and water. The positron produced in this reaction quickly comes to rest, annihilates to $\gamma$-rays, and the $\gamma$-rays Compton scatter to produce fast electrons which are observed in liquid scintillator $\sim 10^{-9}$ seconds after the initial neutrino interaction. The neutron
produced in the initial neutrino interaction collides with protons in the water until it has only thermal energy. When the neutron reaches this point, several microseconds after the initial neutrino interaction, it is captured by the cadmium and γ-rays are produced. Reines and Cowan observed two pulses in the scintillator microseconds apart, the unmistakable signature of a neutrino–proton interaction.

In 1955, Davis, using a reactor as a source, searched and found no evidence for the reaction $\nu + ^{37} Cl \rightarrow ^{37} Ar + e^-$ [5]. Together with the result of Reines and Cowan, Davis’s result showed that the neutrinos produced in a nuclear reactor by the reaction $n \rightarrow pe\nu$ could interact to produce positrons but not electrons. An explanation for this is that lepton number is conserved in interactions and those neutrinos produced in a reactor have the same lepton number as the positron. These experiments provided the first evidence that the neutrino and anti-neutrino were distinct from each other. The “neutrinos” produced in a nuclear reactor are actually anti-neutrinos. Pontecorvo received a mistaken report that Davis had observed the interaction $\nu + ^{37} Cl \rightarrow ^{37} Ar + e^-$. Pontecorvo proposed neutrino–anti-neutrino oscillations [6] as an explanation.

It was not yet known that more than one flavor of neutrino existed or that individual lepton flavor number was conserved. It was known that neutrinos and anti-neutrinos were present in the decays of many particles including $n \rightarrow pe^-\bar{\nu}$, $\pi^- \rightarrow \mu^-\bar{\nu}$, and $\mu^- \rightarrow e^-\nu\bar{\nu}$. It was muon decay, $\mu^- \rightarrow e^-\nu\bar{\nu}$, that provided the first clue about the existence of different neutrino flavors. The branching ratio $\frac{(\mu^- \rightarrow e^-\gamma)}{(\mu^- \rightarrow e^-\nu\bar{\nu})}$ had been calculated independently by both Feinberg [7] and Feynman and Gell-man [8] to be $10^{-4}$. Experiments placed an upper limit of $10^{-8}$ on the branching ratio, much lower than the prediction. This led Lee and Yang to question whether the
neutrino and anti-neutrino produced in muon decay were in fact not a particle-anti-particle pair [9] as had been assumed in the calculations. In 1958, Bludman theorized that two intrinsically different flavors of neutrinos, $\nu_\mu$ coupled to the muon and $\nu_e$ coupled to the electron, existed [10]. These neutrinos each would have their own distinct anti-particles, $\bar{\nu}_\mu$ and $\bar{\nu}_e$, and individual lepton flavor number would be conserved. In this scenario, muons would decay $\mu^- \to e^- \nu_\mu \bar{\nu}_e$. Hence, the neutrinos would not be a particle-anti-particle pair and the decay $\mu^- \to e^- \gamma$ would not be expected to be observed. The present upper limit on the branching ratio is $5 \times 10^{-11}$[11].

In 1962, Lederman, Swartz, and Steinberger et. al. showed that the neutrinos associated with muons produced in pion decay, $\pi^+ \to \mu^+ \nu_\mu$, are different from those neutrinos associated with electrons produced in $\beta$-decay[12]. In this experiment, pions decayed into muons and neutrinos in a decay tunnel, and the muons were stopped in a thick layer of iron at the end of the tunnel. The neutrinos passed through the iron and impinged on a ten ton detector capable of distinguishing muon tracks from electron showers. If only one type of neutrino existed, the neutrino interactions would be expected to produce muons and electrons in nearly equal amounts. In the experiment, muons were produced predominately with no significant number of electrons produced. This result supported the idea of two neutrino flavors and individual lepton flavor conservation. It is now believed that three neutrino flavors exist corresponding to the three known leptons; the electron, the muon, and the tau.

Pontecorvo adapted his neutrino-anti-neutrino oscillation formalism and suggested the possibility of oscillations between neutrinos of different flavors [13]. Independently, Maki et. al. suggested the possibility of neutrino flavor oscillations[14]. Interest in neutrino flavor oscillations increased when Davis measured the $\nu_e$ flux from the sun
to be about one third of the expected value[15]. Neutrino flavor oscillations were proposed as an explanation. Recently, flavor oscillations in matter, not the vacuum oscillations investigated in this document, have been considered as the cause of the solar neutrino discrepancy [16, 17].

For neutrino oscillations to occur, the neutrinos must have non-zero mass and lepton flavor number conservation must be violated. Many theories beyond the standard model predict non-zero neutrino masses and violations of lepton number conservation[18]. Hence, neutrino oscillations give us a tool to look beyond the standard model.

1.2 Phenomenology

The phenomenology of neutrino oscillations is described in detail elsewhere[18, 19, 20, 21]. A brief review follows.

Neutrino oscillations occur if the neutrino states produced in the weak interaction, $\nu_e, \nu_\mu, \nu_\tau$, differ from the neutrino mass eigenstates, $\nu_1, \nu_2, \nu_3$. In this case, the two bases of neutrino states are related at time $t = 0$ by a matrix $U_j$:

$$|\nu_\beta\rangle = \sum_j U_{\beta j} |\nu_j\rangle + \sum_j U'_{\beta j} |N_j\rangle$$

(1.1)

where $\beta = e, \mu, \tau$ and $j = 1, 2, 3$. The $N_j$ are the right-handed isosinglets of heavy neutrinos. For simplicity, these heavy neutrinos are assumed to be essentially decoupled and, therefore, the matrix $U$ is unitary. At time $t > 0$,

$$|\nu_\beta(t)\rangle = \sum_j U_{\beta j} e^{-iE_j t} |\nu_j\rangle$$

(1.2)

where $E_j = \sqrt{p^2 + m_j^2}$. For $E_j \gg m_j, E_j \approx p + m_j^2 / 2p$ and

$$|\nu_\beta(t)\rangle = e^{-i\pi t} \sum_j U_{\beta j} e^{-i m_j^2 t / 2p} |\nu_j\rangle.$$
CHAPTER 1. INTRODUCTION

The probability that a neutrino of flavor $\beta$ at $t = 0$ will have oscillated to a neutrino of flavor $\gamma$ at time $t = t_0 > 0$ is the square of the matrix element between $\nu_\beta$ at $t = 0$ and $\nu_\gamma$ at $t = t_0$.

$$P(\nu_\beta \rightarrow \nu_\gamma) = |\langle \nu_\gamma(t_0)|\nu_\beta(0)\rangle|^2.$$  \hfill (1.4)

$$P(\nu_\beta \rightarrow \nu_\gamma) = |e^{ip_0t_0} \sum_k U_{\gamma k} e^{i\frac{m_k^2 t_0}{2\nu}} \langle \nu_k | \sum_j U_{\beta j} | \nu_j \rangle|^2.$$  \hfill (1.5)

$$P(\nu_\beta \rightarrow \nu_\gamma) = |e^{ip_0t_0} \sum_j U^*_{\gamma j} U_{\beta j} e^{i\frac{m_j^2 t_0}{2\nu}}|^2.$$  \hfill (1.6)

$$P(\nu_\beta \rightarrow \nu_\gamma) = (\sum_j U^*_{\gamma j} U_{\beta j} e^{i\frac{m_j^2 t_0}{2\nu}})(\sum_k U_{\gamma k} U^*_{\beta k} e^{-i\frac{m_k^2 t_0}{2\nu}}).$$  \hfill (1.7)

$$P(\nu_\beta \rightarrow \nu_\gamma) = \sum_j |U^*_{\gamma j} U_{\beta j}|^2 + 2 \sum_{j<k} Re(U^*_{\gamma j} U_{\beta j} U_{\gamma k} U^*_{\beta k} e^{i(m_j^2 - m_k^2) t_0/2\nu}).$$  \hfill (1.8)

If $c = 1$, the distance travelled $L$ is equal to the time $t_0$. The quantity $\Delta_{jk}$ is also defined:

$$\Delta_{jk} = \frac{1}{2}(m_k^2 - m_j^2)(\frac{L}{2\nu}) = \frac{\delta m^2_{jk} L}{4 \nu}.$$  \hfill (1.9)

In conventional units,

$$\Delta_{jk} = \frac{1.27 \delta m^2_{jk} (e V^2) L(m)}{E(MeV)}.$$  \hfill (1.10)

Now,

$$\sum_j |U^*_{\gamma j} U_{\beta j}|^2 = \sum_j U^*_{\gamma j} U_{\beta j}^2 - 2 \sum_{j<k} Re(U^*_{\gamma j} U_{\beta j} U_{\gamma k} U^*_{\beta k}),$$  \hfill (1.11)

and since the eigenstates are orthogonal,

$$\sum_j |U^*_{\gamma j} U_{\beta j}|^2 = -2 \sum_{j<k} Re(U^*_{\gamma j} U_{\beta j} U_{\gamma k} U^*_{\beta k}).$$  \hfill (1.12)

Therefore,

$$P(\nu_\beta \rightarrow \nu_\gamma) = -2 \sum_{j<k} Re(U^*_{\gamma j} U_{\beta j} U_{\gamma k} U^*_{\beta k})$$

$$+ 2 \sum_{j<k} Re(U^*_{\gamma j} U_{\beta j} U_{\gamma k} U^*_{\beta k} e^{i(m_j^2 - m_k^2) t_0/2\nu}).$$  \hfill (1.13)
\begin{align}
P(v_\beta \to v_\gamma) &= -2 \sum_{j<k} \text{Re}(U_{\gamma j}^* U_{\beta j} U_{\gamma k} U_{\beta k}^*) \\
&\quad + 2 \sum_{j<k} \text{Re}(U_{\gamma j}^* U_{\beta j} U_{\gamma k} U_{\beta k}^* e^{-2i\Delta_{jk}}). \\
(1.14) \\

P(v_\beta \to v_\gamma) &= -2 \sum_{j<k} \text{Re}(U_{\gamma j}^* U_{\beta j} U_{\gamma k} U_{\beta k}^*) \\
&\quad + 2 \sum_{j<k} \text{Re}(U_{\gamma j}^* U_{\beta j} U_{\gamma k} U_{\beta k}^*) \cos(2\Delta_{jk}) \\
&\quad + 2 \sum_{j<k} \text{Im}(U_{\gamma j}^* U_{\beta j} U_{\gamma k} U_{\beta k}^*) \sin(2\Delta_{jk}). \\
(1.15) \\

P(v_\beta \to v_\gamma) &= +2 \sum_{j<k} \text{Re}(U_{\gamma j}^* U_{\beta j} U_{\gamma k} U_{\beta k}^*) (\cos(2\Delta_{jk}) - 1) \\
&\quad + 2 \sum_{j<k} \text{Im}(U_{\gamma j}^* U_{\beta j} U_{\gamma k} U_{\beta k}^*) \sin(2\Delta_{jk}). \\
(1.16) \\

P(v_\beta \to v_\gamma) &= -4 \sum_{j<k} \text{Re}(U_{\gamma j}^* U_{\beta j} U_{\gamma k} U_{\beta k}^*) \sin^2 \Delta_{jk} \\
&\quad + 2 \sum_{j<k} \text{Im}(U_{\gamma j}^* U_{\beta j} U_{\gamma k} U_{\beta k}^*) \sin(2\Delta_{jk}). \\
(1.17)
\end{align}

The actual form of the probability depends on the form of \( U \). \( U \) is a 3 \times 3 unitary matrix characterized by nine real parameters and six phases. If \( U \) is real, unitarity requires \( U^TU = 1 \) and only three real parameters remain. Five of the phases can be eliminated and the remaining phase is responsible for CP violation. \( U \) is real if CP is conserved.

There are two commonly used parameterizations of \( U \). One is the Kobayashi-Maskawa parameterization:

\[
\begin{pmatrix}
  c_1 & -s_1c_3 & -s_1s_3 \\
  s_1c_2 & c_1c_2c_3 - s_2c_6e^{i\delta} & c_1c_2s_3 + s_2c_6e^{i\delta} \\
  s_1s_2 & c_1s_2c_3 + c_2s_6e^{i\delta} & c_1s_2s_3 - c_2c_6e^{i\delta}
\end{pmatrix}
\]

\[ (1.18) \]

The other is the Maiani parameterization:

\[
\begin{pmatrix}
c_\beta c_\delta & c_\beta s_\delta \\
-s_\gamma s_\beta s_\delta - s_\delta c_\delta & c_\gamma c_\delta - s_\gamma s_\delta e^{i\delta} \\
-s_\beta c_\gamma c_\delta - s_\delta s_\gamma e^{-i\delta} & -c_\gamma s_\beta s_\delta - s_\gamma c_\delta e^{i\delta}
\end{pmatrix}
\]

\[ (1.19) \]
In both parameterizations, \( s \equiv \sin, c \equiv \cos \), and \( \delta = 0 \) if CP is conserved.

The Maiani parameterization has the advantage that the angles \( \theta, \beta, \gamma \) are directly related to the mixing amplitudes of pairs of neutrino flavors, i.e.,

\[
\begin{align*}
\theta & \rightarrow \alpha_{e\mu} \\
\beta & \rightarrow \alpha_{e\tau} \\
\gamma & \rightarrow \alpha_{\mu\tau}.
\end{align*}
\]

This decoupling does not take place with the Kobayashi-Maskawa matrix.

For simplicity, results of oscillation experiments are customarily presented assuming oscillations occur between only two flavors and CP is conserved. Three generation analysis has been done [22] but is much more complex. In the two flavor case, with CP conserved,

\[
U = \begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix},
\]

and

\[
P(\nu_{\mu} \rightarrow \nu_{e}) = 4 \sin^{2} \theta \cos^{2} \theta \sin^{2} \Delta_{12} = \sin^{2}(2\theta) \sin^{2} \left( \frac{1.27 \delta m_{12}^{2}(cV^{2})L(m)}{E(MeV)} \right).
\]

In any neutrino oscillation experiment, choices must be made about the distance from the neutrino source to the detector, \( L \), and the beam energy, \( E \). Larger values of the ratio \( L/E \) maximize the sensitivity to \( \delta m^{2} \), but it is also desirable to maximize the number of neutrino interactions in the detector to increase sensitivity to \( \sin^{2}(2\theta) \) and minimize statistical error. Unfortunately, the neutrino flux falls with increased \( L \) and the neutrino cross section rises with increased \( E \). A large \( L/E \) ratio requires a large value of \( L \) and small value of \( E \), while a small value of \( L \) and large value of \( E \) increase the number of neutrino interactions. Any neutrino oscillation experiment
faces this tradeoff. In E776, the detector is 1 km from the neutrino source and the neutrino flux peaks at energies between 1 and 2 GeV.

1.3 Experimental Overview

E776 searched for neutrino oscillations by searching for the appearance of $\nu_e$'s in a beam of $\nu_{\mu}$'s. Neutrinos cannot be detected directly. Therefore, a large mass detector served as a target for the neutrinos to interact and the interaction products were detected. In a perfect world, we would start with a pure beam of $\nu_{\mu}$'s and have a detector capable of perfectly distinguishing the products of a $\nu_{\mu}$ interaction from the products of a $\nu_e$ interaction and simply look for the products of a $\nu_e$ interaction appearing in our detector. In practice, the $\nu_{\mu}$ beam is not pure and contains $\nu_e$'s, $\bar{\nu}_{\mu}$'s, and $E_{e}$'s. Another problem is that the products of a $\nu_{\mu}$ interaction can look like the products of a $\nu_e$ interaction in our detector. We attempt to solve these problems using a Monte Carlo simulation to predict the fluxes of the various neutrino flavors through our detector (assuming no oscillations), what type of interactions (if any) those neutrinos participate in, and what the products of those interactions look like in our detector. The experimental data and Monte Carlo are then analyzed identically to find quasi-elastic $\nu_{\mu}$ interactions, $\nu_{\mu}n \rightarrow p\mu^{-}$, and quasi-elastic $\nu_e$ interactions, $\nu_{e}n \rightarrow p\epsilon^{-}$.

We look for quasi-elastic interactions because the neutrino energy can be reconstructed, using only the energy and angle of the lepton produced, by the formula

$$E_{\nu} = \frac{2E_{l}m_{\bar{p}} - m_{l}^{2}}{2m_{p} - 2E_{l} + 2p_{l}\cos\theta}$$  \hspace{1cm} (1.22)$$

where $E_{\nu}$ is the neutrino energy, $E_{l}$ is the lepton energy, $m_{\bar{p}}$ is the proton mass, $m_{l}$ is the lepton mass, $p_{l}$ is the lepton momentum, and $\theta$ is the angle between the lepton
momentum direction and the beam direction.

We can set limits on the neutrino oscillation parameters, $\delta m^2$ and $\sin^2(2\theta)$, by comparing the final sample of actual $\nu_e$ events with the Monte Carlo prediction for the final $\nu_e$ sample. The muon neutrino analysis checks and normalizes the Monte Carlo prediction. Not all events in the final $\nu_e$ sample are actually quasi-elastic neutrino events, but the Monte Carlo prediction also contains this misidentification. We would expect to see an excess of $\nu_e$ events in the data compared to the Monte Carlo prediction if $\nu_\mu$'s oscillated into $\nu_e$'s. Since no such excess is observed, we can only set limits on the oscillation parameters. The chapters that follow provide details about the apparatus, analysis, and results of E776.
Chapter 2

Neutrino Beam

This experiment, E776, was performed at Brookhaven National Laboratory (BNL), using the wide band neutrino beam from the Alternating Gradient Synchrotron (AGS). This beam is produced when protons from the AGS are extracted and transported to impinge on a titanium target. Interactions between the protons and target nuclei produce a secondary beam of charged mesons. A magnetic horn system focuses and charge selects the mesons over a wide momentum range. The mesons decay in flight in the decay tunnel to produce neutrinos and other particles. Particles other than neutrinos are stopped by steel shielding at the end of the decay tunnel. Beam monitors were placed in the proton beam, decay tunnel, and steel shielding.

Figure 2.1 is an overview of the laboratory showing the location of the AGS proton ring, proton extraction line, meson decay tunnel, and the E776 detector.

Protons are boosted to 200 $MeV/c$ by the linear accelerator injected into the AGS, and accelerated to 28.3 $GeV/c$. The AGS beam spill lasts 2.7 $\mu sec$ and has an RF structure of twelve bunches, of width $\sim 35$ $nsec$ separated by 224 $nsec$ as shown in Figure 2.2. Every 1.4 sec, the protons are extracted during a single revolution, preserving the RF structure, into the U-line for transport to the target. Sometimes,
Figure 2.1: Overview of the AGS, beam line, and detector site.
one of the bunches was diverted for use by other AGS experiments.

The protons travel 730 feet in the U-line to reach the target. A series of dipole and quadrupole magnets along the entire U-line focus and guide the proton beam. There is a 4° bend after 50 feet and an 8° bend after 290 feet. The bends reduce the possibility of background from proton scraping.

The proton beam intensity is measured by three current transformers. The first, XCBM, measures proton intensity in the AGS main ring before extraction. The other transformers, U15 and U716, located 15 feet and 716 feet after the extraction point (respectively), measure the proton intensity in the U-line. There are also 32 radiation monitors along the U-line to measure beam losses. Since it is closest to the target,
CHAPTER 2. NEUTRINO BEAM

U716 is used to provide an estimate of the number of protons hitting the target. Typically, $1.3 \times 10^{13}$ protons hit the target during each beam spill.

The target is a titanium cylinder 50.8 cm long with radius 6.4 mm. For 28 GeV/c protons, this is about 1.9 interaction lengths. The target is divided in ten equal segments pressed into an aluminum sheath. The sheath is evacuated, filled with one atmosphere of helium, and sealed with aluminum plugs on each end. The helium pressure is monitored to detect target disintegration. The protons interact in the target to produce pions, kaons and other particles.

The target assembly is embedded in a magnetic horn focusing system which focuses and selects the charged particles which emerge from the target. For the run analyzed in this document, positively charged particles were selected and focused. Two cylindrically symmetric aluminum conductors producing a toroidal magnetic field which varies inversely with distance from the center of symmetry make up the horn system. The upstream horn is 213 cm long and the downstream horn is 152 cm long. The entire horn system is about 10 m long and is shown in Figure 2.3. The horn is designed to focus all positive particles with no momentum selection to maximize neutrino flux. Not all negative particles are defocused. Some negative particles have very high energy or small initial angle and get through the horn system. Some negative particles decay before the horn system. These particles produce an antineutrino background in the neutrino beam.

The charged mesons which make up the secondary beam decay producing neutrinos in a 38 m decay tunnel shown in Figure 2.4. The tunnel contains two pion monitors and three lucite Čerenkov counters along with the target and horn system. The Čerenkov counters provide accurate beam timing to eliminate cosmic ray background.
Figure 2.3: The WBB magnetic horn system.
The pion monitors [23] measure the halo of the secondary beam. The monitors were not placed directly in the beam because the high intensity would have destroyed them.

The decay products other than the neutrinos, along with the undecayed mesons of the secondary beam, are stopped by 30 m of steel at the end of the decay tunnel. The beam stop contains muon counters to monitor the beam profile. These counters only measure muons with energy higher than 4 GeV.
Figure 2.4: The WBB decay tunnel.
Chapter 3

Detector

The E776 detector consists of a large fiducial mass electromagnetic calorimeter followed by a muon spectrometer downstream. The calorimeter provides target material for neutrino interactions and measures electromagnetic shower energies, particle trajectories, and event times. The muon spectrometer is made of magnetized steel for measuring muon momentum and charge. The detector is shown in Figure 3.1. Not shown in the figure are a 5 cm thick lead wall immediately upstream of the detector and a ~1.5 m thick concrete wall 5 m upstream of the detector. The concrete wall shields the detector from neutrons and muons in the beam and the lead converts gamma rays into showers before they enter the detector. More detailed descriptions of the detector can be found elsewhere [24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35].

3.1 Electromagnetic Calorimeter

The electromagnetic calorimeter is made of ninety planes of proportional drift tubes (PDTs) alternating with 1 in thick (1/4 radiation length) concrete absorbers with a total mass of ~230 metric tons. Consecutive planes of drift tubes are placed orthogonally to give position information about both transverse directions. Every
Figure 3.1: The E775 neutrino detector.
tenth plane of concrete is replaced by a plane of acrylic scintillator for event timing.

The planes of PDTs in the calorimeter are numbered 0 to 89 with plane 89 being the most upstream. The even-numbered planes have horizontal wires and give us information about the vertical position of tracks and are called y planes. The odd-numbered planes have vertical wires and give us information about the horizontal position of tracks and are called x planes. The PDTs within each plane are numbered from 0 to 63. For x planes, PDT 0 is on the right of someone standing at the upstream end of the detector facing the detector. For y planes, PDT 0 is on the bottom of the detector.

Each plane of PDTs is composed of 16 18 ft × 13.54 in × 1.74 in extruded aluminum chambers. Each chamber contains four cells separated by a 0.1 in wall. Thus, each plane contains 64 cells with interior dimensions 18 ft × 3.25 in × 1.5 in. The corners of each cell are filled in as shown in Figure 3.1 to eliminate regions of high drift time and increase the strength of the tubes. A 50 μm gold plated tungsten wire in the center of each cell acts as both field and sense wire. The wire is at 2200 V and the cell walls are grounded to give large pulses while running in proportional mode. The tubes are filled with a gas mixture of 80% argon and 20% ethane. The gas flows slowly through the detector to eliminate impurities that might leak in.

The concrete used in the detector has an average density of ~ 2.3 g/cm³ and an average Z of about 10. Concrete was chosen as the absorber in the detector because of its strength, relatively low cost, and medium Z. The radiation length is approximately[36]

\[
\frac{1}{L_{\text{rad}}} = \frac{4N_A Z^2 \tau_e}{137 A} \left( \ln \frac{183}{Z^{1/3}} \right),
\]

(3.1)

with \(N_A\) Avagadro's number and \(\tau_e\) the classical radius of an electron. The radiation
length per target mass goes as $1/Z^2$. Concrete doesn't have a high number of target nucleons but does allow good shower development and energy resolution.

Each scintillator plane was constructed of 8 sections of acrylic plastic, each 100 in $\times$ 50 in $\times$ 1.0 in as shown in Figure 3.2. Each section was surrounded on three sides by waveshifter bars to direct the light to the fourteen phototubes (PMTs) for each plane. The scintillator and waveshifter were wrapped in light tight plastic and encased in an aluminum housing. The thickness of the aluminum was such that the aluminum and scintillator together are $\sim 1/4$ of radiation length in thickness, the same as a plane of concrete absorber.

### 3.2 Muon Spectrometer

Immediately downstream of the calorimeter is a muon spectrometer. The most upstream part of the spectrometer is two orthogonal planes of PDTs rotated 45° with respect to the vertical and horizontal PDT planes of the calorimeter. These are followed by five toroidally magnetized iron plates. The three most upstream plates are 5 in thick while the two most downstream plates are 7 in thick. A 15 kA current passes through the windings around the plates creating a magnetic field of 18 kG near the center which falls off to $\sim 15$ kG at the edge. The field is such that negatively charged particles are focused. Following each plate is a pair of orthogonal PDT planes with the same orientations as the PDT planes of the calorimeter. Following the downstream plate, four addition PDT planes are added for a total of 6 planes following that plate. Alternate planes are orthogonal. There is 20.75 in between the last iron plate and the six PDT planes to give a long lever arm to measure the angle at which tracks exit. The PDTs in the spectrometer are the same as those in the calorimeter.
Figure 3.2: A scintillator plane.
except their lengths are adjusted to fit the toroid shape and to accommodate the hole where the current windings passed.
Chapter 4

Monte Carlo

An extensive Monte Carlo program was written for E776. The Monte Carlo predicts how many of each kind of neutrino interaction will occur in our detector and what the products of those interactions will look like in our detector if no oscillations occur. Using the Monte Carlo, we can determine if those events we identify as being $\nu_e$ interactions can be accounted for by beam contamination or misidentified $\nu_\mu$ interactions. An excess of $\nu_e$ events in the data compared to the Monte Carlo prediction would be evidence of neutrino oscillations. It is not desirable that an experimental result depend so heavily on a Monte Carlo prediction. But with alternative methods of determining the number of $\nu_e$ events that we expect if oscillations do no occur (such as a second detector at a different location) being impractical, the Monte Carlo is a necessary evil.

The E776 Monte Carlo is actually two Monte Carlos. The first, the beam Monte Carlo\cite{30, 34}, predicts the flux of the various neutrino flavors through the detector. The second, the event Monte Carlo\cite{32}, predicts how many neutrinos of each flavor will interact in the detector and what those events will look like.
4.1 Beam Monte Carlo

An extensive beam Monte Carlo was developed [30] to calculate the fluxes and spectra of $\nu_\mu$, $\nu_\tau$, $\bar{\nu}_\mu$, and $\bar{\nu}_\tau$. The first part of the beam Monte Carlo simulates the production of secondary hadrons resulting from collisions between the proton beam and the titanium target. The proton beam is modeled as a monoenergetic 28.3 GeV parallel beam. The transverse distribution of the beam is modeled by a bivariate gaussian of 1.8 mm full width at half maximum centered along the target axis. The simulation of the interaction of the proton beam and the target includes nuclear elastic and inelastic interactions, ionization losses, Coulomb scattering, particle reinteraction, and particle absorption in the target and assembly.

Hadron production in proton beam-target nuclei collisions is simulated using two models, EVQ and GHR. EVQ is a composite model which simulates proton-nucleus collisions using NUCEVT[37] for incident particles of momenta greater than 5 GeV/c and NUCRIN[38] for incident particles of lower momenta. NUCEVT is a multi-chain fragmentation model and NUCRIN is a combination of a resonance production model and a parameterization of the emission of cascade nucleons from intranuclear collisions. GHR is a parameterization by Grote, Hagendorn, and Ranft[39]. All hadron production is simulated using EVQ except for strange particle production where GHR is used. GHR predicts a higher $K^+/\pi^+$ ratio than EVQ does. The $\nu_\mu$ beam contamination depends on the $K^+/\pi^+$ ratio and GHR is used for strange particle production to lessen the chance of an underestimation of this contamination.

The GHR parameterization uses a thin copper target and it must be modified to take into account the thick titanium target used in this experiment. The GHR
<table>
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<th>Particle</th>
<th>#/POT</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+$</td>
<td>1.12</td>
<td>EVQ</td>
</tr>
<tr>
<td>$\pi^-$</td>
<td>0.947</td>
<td>EVQ</td>
</tr>
<tr>
<td>$K^+$</td>
<td>0.121</td>
<td>GHR</td>
</tr>
<tr>
<td>$K^-$</td>
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</tr>
<tr>
<td>$K^0$</td>
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<td>GHR</td>
</tr>
<tr>
<td>$\bar{K}^0$</td>
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<td>GHR</td>
</tr>
<tr>
<td>$\Lambda^0$</td>
<td>0.07</td>
<td>GHR</td>
</tr>
</tbody>
</table>

Table 4.1: The target production integrated over 0 to 19 degrees polar angle and momentum of 0 to 28 GeV/c.

The parameterization was scaled using the following formula

$$P_{\text{thick}}^{GHR}(T) = P_{\text{thin}}^{GHR}(Cu) \times \frac{P_{\text{thick}}^{EVQ}(T)}{P_{\text{thin}}^{EVQ}(Cu)} \quad (4.1)$$

where $P_{\tau}^{\varphi}(\eta)$ represents particle production generated by model $\varphi$, target thickness $\tau$, and target material $\eta$. The production of hadrons integrated over polar angles from 0° to 19° and momenta from 0 GeV/c to 28 GeV/c are listed in Table 4.1. The maximum angle accepted by the first horn is 19°.

The Monte Carlo then simulates the transport of the hadrons through the horn system and decay tunnel. The hadrons are either focused or defocused by the magnetic field. They may also be absorbed by the horn elements or walls. The hadrons may decay anytime after they are produced until they reach the steel shielding where undecayed particles are absorbed. The daughter particles may also decay until they reach the shield. The decay chains included in the Monte Carlo which produce the various neutrino flavors are listed in Table 4.2, Table 4.3, Table 4.4, and Table 4.5.

The neutrinos produced in these decays are assumed not to interact or oscillate. Those neutrinos whose trajectories pass through the neutrino detector contribute to the neutrino flux. The dominant contributions to the flux of the various flavors are
1. $\pi^+ \rightarrow \mu^+ \nu_{\mu}$  
2. $K^+ \rightarrow \mu^+ \nu_{\mu}$  
3. $K^+ \rightarrow \pi^0 \pi^- \rightarrow \mu^+ \nu_{\mu}$  
4. $K^+ \rightarrow \pi^0 \mu^+ \nu_{\mu}$  
5. $\pi^- \rightarrow \mu^- \bar{\nu}_{\mu} \rightarrow e^- \nu_{\mu} \bar{\nu}_{e}$  
6. $K^- \rightarrow \mu^- \bar{\nu}_{\mu} \rightarrow e^- \nu_{\mu} \bar{\nu}_{e}$  
7. $K^- \rightarrow \pi^0 \mu^- \bar{\nu}_{\mu} \rightarrow e^- \nu_{\mu} \bar{\nu}_{e}$  
8. $K^- \rightarrow \pi^0 \mu^- \bar{\nu}_{\mu} \rightarrow e^- \nu_{\mu} \bar{\nu}_{e}$  
9. $K^0_S \rightarrow \pi^- \pi^+ \rightarrow \mu^+ \nu_{\mu}$  
10. $K^0_S \rightarrow \pi^- \pi^+ \rightarrow \mu^+ \nu_{\mu}$  
11. $K^0_L \rightarrow \pi^+ e^- \bar{\nu}_{e} \rightarrow \mu^+ \nu_{\mu}$  
12. $K^0_L \rightarrow \pi^- \mu^+ \nu_{\mu}$  
13. $K^0_L \rightarrow \pi^- \mu^+ \nu_{\mu}$  
14. $K^0_L \rightarrow \pi^+ e^- \nu_{e} \rightarrow \mu^+ \nu_{\mu}$  
15. $K^0_L \rightarrow \pi^- e^- \nu_{e} \rightarrow \mu^+ \nu_{\mu}$  
16. $K^0_L \rightarrow \pi^- \mu^+ \nu_{\mu} \rightarrow \mu^+ \nu_{\mu}$  
17. $K^0_L \rightarrow \pi^0 \pi^- \pi^+ \rightarrow \mu^+ \nu_{\mu}$  
18. $K^0_L \rightarrow \pi^0 \pi^- \pi^+ \rightarrow \mu^+ \nu_{\mu}$  

Table 4.2: The decay modes included in the Monte Carlo for the calculation of the $\nu_{\mu}$ flux.

listed in Table 4.6, Table 4.7, Table 4.8, and Table 4.9. The neutrino spectra for the various flavors are shown in Figure 4.1. The dominant source of uncertainty in the beam calculation is the uncertainty in the inelastic cross section for proton-titanium nuclei collisions. The systematic uncertainty in $\nu_{\mu}$ flux is estimated to be $\sim 10\%$. The $\nu_e / \nu_{\mu}$ ratio is estimated to be 0.0062 with an uncertainty of $\sim 10\%$ due to uncertainties in the $K/\pi$ ratio.

4.2 Event Monte Carlo

The event Monte Carlo begins by choosing the energy of the interacting neutrino according to the predicted energy spectrum of interacting neutrinos of the desired flavor. This spectrum is the product of the beam Monte Carlo flux prediction and
1. \( \pi^+ \rightarrow \mu^+ \nu_\mu \rightarrow e^+ \overline{\nu}_\mu \nu_\bar{e} \)
2. \( \pi^+ \rightarrow e^+ \nu_\bar{e} \)
3. \( K^+ \rightarrow \pi^0 e^+ \nu_\bar{e} \)
4. \( K^+ \rightarrow \mu^+ \nu_\mu \rightarrow e^+ \overline{\nu}_\mu \nu_\bar{e} \)
5. \( K^+ \rightarrow \pi^0 \mu^+ \nu_\mu \rightarrow e^+ \overline{\nu}_\mu \nu_\bar{e} \)
6. \( K^+ \rightarrow \pi^0 \pi^+ \rightarrow \mu^+ \nu_\mu \rightarrow e^+ \overline{\nu}_\mu \nu_\bar{e} \)
7. \( K_S^0 \rightarrow \pi^- \pi^+ \rightarrow e^+ \nu_\bar{e} \)
8. \( K_S^0 \rightarrow \pi^- \pi^+ \rightarrow \mu^+ \nu_\mu \rightarrow e^+ \overline{\nu}_\mu \nu_\bar{e} \)
9. \( K_L^0 \rightarrow \pi^- e^+ \nu_\bar{e} \)
10. \( K_L^0 \rightarrow \pi^- e^- \nu_\bar{e} \rightarrow e^+ \nu_\bar{e} \)
11. \( K_L^0 \rightarrow \pi^- e^- \nu_\bar{e} \rightarrow e^+ \nu_\bar{e} \)
12. \( K_L^0 \rightarrow \pi^- e^- \nu_\bar{e} \rightarrow e^+ \nu_\bar{e} \)
13. \( K_L^0 \rightarrow \pi^- e^- \nu_\bar{e} \rightarrow e^+ \nu_\bar{e} \)
14. \( K_L^0 \rightarrow \pi^- e^- \nu_\bar{e} \rightarrow e^+ \nu_\bar{e} \)
15. \( K_L^0 \rightarrow \pi^- e^- \nu_\bar{e} \rightarrow e^+ \nu_\bar{e} \)
16. \( K_L^0 \rightarrow \pi^- e^- \nu_\bar{e} \rightarrow e^+ \nu_\bar{e} \)

Table 4.3: The decay modes included in the Monte Carlo for the calculation of the \( \nu_\bar{e} \) flux.

1. \( \pi^- \rightarrow e^- \overline{\nu}_\bar{e} \)
2. \( \pi^- \rightarrow \mu^- \overline{\nu}_\bar{\mu} \rightarrow e^- \nu_\mu \overline{\nu}_\bar{e} \)
3. \( K^- \rightarrow \pi^0 e^- \overline{\nu}_\bar{e} \)
4. \( K^- \rightarrow \mu^- \overline{\nu}_\bar{\mu} \rightarrow e^- \nu_\mu \overline{\nu}_\bar{e} \)
5. \( K^- \rightarrow \pi^0 \mu^- \overline{\nu}_\bar{\mu} \rightarrow e^- \nu_\mu \overline{\nu}_\bar{e} \)
6. \( K^- \rightarrow \pi^0 \mu^- \overline{\nu}_\bar{\mu} \rightarrow e^- \nu_\mu \overline{\nu}_\bar{e} \)
7. \( K_S^0 \rightarrow \pi^- e^- \nu_\bar{e} \)
8. \( K_S^0 \rightarrow \pi^- e^- \nu_\bar{e} \rightarrow \mu^- \overline{\nu}_\bar{\mu} \rightarrow e^- \nu_\mu \overline{\nu}_\bar{e} \)
9. \( K_L^0 \rightarrow \pi^- e^- \nu_\bar{e} \)
10. \( K_L^0 \rightarrow \pi^- e^- \nu_\bar{e} \rightarrow \mu^- \overline{\nu}_\bar{\mu} \rightarrow e^- \nu_\mu \overline{\nu}_\bar{e} \)
11. \( K_L^0 \rightarrow \pi^- e^- \nu_\bar{e} \rightarrow \mu^- \overline{\nu}_\bar{\mu} \rightarrow e^- \nu_\mu \overline{\nu}_\bar{e} \)
12. \( K_L^0 \rightarrow \pi^- e^- \nu_\bar{e} \rightarrow \mu^- \overline{\nu}_\bar{\mu} \rightarrow e^- \nu_\mu \overline{\nu}_\bar{e} \)
13. \( K_L^0 \rightarrow \pi^- e^- \nu_\bar{e} \rightarrow \mu^- \overline{\nu}_\bar{\mu} \rightarrow e^- \nu_\mu \overline{\nu}_\bar{e} \)
14. \( K_L^0 \rightarrow \pi^- e^- \nu_\bar{e} \rightarrow \mu^- \overline{\nu}_\bar{\mu} \rightarrow e^- \nu_\mu \overline{\nu}_\bar{e} \)
15. \( K_L^0 \rightarrow \pi^- e^- \nu_\bar{e} \rightarrow \mu^- \overline{\nu}_\bar{\mu} \rightarrow e^- \nu_\mu \overline{\nu}_\bar{e} \)
16. \( K_L^0 \rightarrow \pi^- e^- \nu_\bar{e} \rightarrow \mu^- \overline{\nu}_\bar{\mu} \rightarrow e^- \nu_\mu \overline{\nu}_\bar{e} \)
17. \( \Lambda^0 \rightarrow p e^- \overline{\nu}_\bar{e} \)

Table 4.4: The decay modes included in the Monte Carlo for the calculation of the \( \overline{\nu}_e \) flux.
1. $\pi^- \rightarrow \mu^- \bar{\nu}_\mu$
2. $K^- \rightarrow \mu^- \bar{\nu}_\mu$
3. $K^- \rightarrow \pi^0 \pi^- \rightarrow \mu^- \bar{\nu}_\mu$
4. $K^- \rightarrow \pi^0 \mu^- \bar{\nu}_\mu$
5. $\pi^+ \rightarrow \mu^+ \nu_\mu \rightarrow e^+ \bar{\nu}_\mu \nu_e$
6. $K^+ \rightarrow \mu^+ \nu_\mu \rightarrow e^+ \bar{\nu}_\mu \nu_e$
7. $K^+ \rightarrow \pi^0 \mu^+ \nu_\mu \rightarrow e^+ \bar{\nu}_\mu \nu_e$
8. $K^+ \rightarrow \pi^0 \pi^+ \rightarrow \mu^+ \nu_\mu \rightarrow e^+ \bar{\nu}_\mu \nu_e$
9. $K^0_S \rightarrow \pi^- \pi^+ \rightarrow \mu^- \bar{\nu}_\mu$
10. $K^0_S \rightarrow \pi^- \pi^+ \rightarrow \mu^+ \nu_\mu \rightarrow e^+ \bar{\nu}_\mu \nu_e$
11. $K^0_L \rightarrow \pi^+ \mu^- \bar{\nu}_\mu$
12. $K^0_L \rightarrow \pi^- e^+ \nu_e \rightarrow \mu^- \bar{\nu}_\mu$
13. $K^0_L \rightarrow \pi^- e^+ \nu_e \rightarrow \mu^- \bar{\nu}_\mu$
14. $K^0_L \rightarrow \pi^+ \mu^- \bar{\nu}_\mu \rightarrow e^+ \bar{\nu}_\mu \nu_e$
15. $K^0_L \rightarrow \pi^- e^- \bar{\nu}_e \rightarrow \mu^- \bar{\nu}_\mu \rightarrow e^+ \bar{\nu}_\mu \nu_e$
16. $K^0_L \rightarrow \pi^+ \mu^- \bar{\nu}_\mu \rightarrow e^+ \bar{\nu}_\mu \nu_e$
17. $K^0_L \rightarrow \pi^0 \pi^- \pi^+ \rightarrow \mu^- \bar{\nu}_\mu$
18. $K^0_L \rightarrow \pi^0 \pi^- \pi^+ \rightarrow \mu^+ \nu_\mu \rightarrow e^+ \bar{\nu}_\mu \nu_e$

Table 4.5: The decay modes included in the Monte Carlo for the calculation of the $\bar{\nu}_\mu$ flux.

<table>
<thead>
<tr>
<th>Source</th>
<th>Flux</th>
<th>% of Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+ \rightarrow \mu^+ \nu_\mu$</td>
<td>21.9</td>
<td>94</td>
</tr>
<tr>
<td>$K^+ \rightarrow \mu^+ \nu_\mu$</td>
<td>0.73</td>
<td>3.1</td>
</tr>
<tr>
<td>$K^0_S \rightarrow \pi^- \pi^+ \rightarrow \mu^+ \nu_\mu$</td>
<td>0.57</td>
<td>2.4</td>
</tr>
<tr>
<td>$K^+ \rightarrow \pi^0 \pi^+ \rightarrow \mu^+ \nu_\mu$</td>
<td>0.05</td>
<td>0.2</td>
</tr>
<tr>
<td>$K^+ \rightarrow \pi^0 \mu^+ \nu_\mu$</td>
<td>0.04</td>
<td>0.2</td>
</tr>
<tr>
<td>total</td>
<td>23.30</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 4.6: Integrated contributions to the $\nu_\mu$ flux in $\nu/m^2/10^6 POT$. 
### Table 4.7: Integrated contributions to the $\nu_\epsilon$ flux in $\nu/m^2/10^6 POT$.

<table>
<thead>
<tr>
<th>Source</th>
<th>Flux</th>
<th>% of Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^+ \rightarrow \pi^0 e^+ \nu_\epsilon$</td>
<td>0.064</td>
<td>44</td>
</tr>
<tr>
<td>$\pi^+ \rightarrow \mu^+ \nu_\mu \rightarrow e^+ \overline{\nu}<em>\mu \nu</em>\epsilon$</td>
<td>0.060</td>
<td>42</td>
</tr>
<tr>
<td>$K_L^0 \rightarrow \pi^- e^+ \nu_\epsilon$</td>
<td>0.014</td>
<td>10</td>
</tr>
<tr>
<td>$\pi^+ \rightarrow e^+ \nu_\epsilon$</td>
<td>0.003</td>
<td>2</td>
</tr>
<tr>
<td>$K^+ \rightarrow \mu^+ \nu_\mu \rightarrow e^+ \overline{\nu}<em>\mu \nu</em>\epsilon$</td>
<td>0.002</td>
<td>1</td>
</tr>
<tr>
<td>$K^0_S \rightarrow \pi^- \pi^+ \rightarrow \mu^+ \nu_\mu \rightarrow e^+ \overline{\nu}<em>\mu \nu</em>\epsilon$</td>
<td>0.002</td>
<td>1</td>
</tr>
<tr>
<td><strong>total</strong></td>
<td>0.144</td>
<td>100</td>
</tr>
</tbody>
</table>

### Table 4.8: Integrated contributions to the $\nu_\mu$ flux in $\nu/m^2/10^6 POT$.

<table>
<thead>
<tr>
<th>Source</th>
<th>Flux</th>
<th>% of Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^- \rightarrow \mu^- \overline{\nu}_\mu$</td>
<td>0.709</td>
<td>84</td>
</tr>
<tr>
<td>$\pi^+ \rightarrow \mu^+ \nu_\mu \rightarrow e^+ \overline{\nu}<em>\mu \nu</em>\epsilon$</td>
<td>0.060</td>
<td>7</td>
</tr>
<tr>
<td>$K^- \rightarrow \mu^- \overline{\nu}_\mu$</td>
<td>0.034</td>
<td>4</td>
</tr>
<tr>
<td>$K^0_L \rightarrow \pi^- \pi^+ \rightarrow \mu^- \overline{\nu}_\mu$</td>
<td>0.016</td>
<td>2</td>
</tr>
<tr>
<td>$K^0_L \rightarrow \pi^- \mu^- \overline{\nu}_\mu$</td>
<td>0.010</td>
<td>1</td>
</tr>
<tr>
<td><strong>total</strong></td>
<td>0.841</td>
<td>100</td>
</tr>
</tbody>
</table>

### Table 4.9: Integrated contributions to the $\overline{\nu}_\epsilon$ flux in $\nu/m^2/10^6 POT$.

<table>
<thead>
<tr>
<th>Source</th>
<th>Flux</th>
<th>% of Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^0_L \rightarrow \pi^- e^- \nu_\epsilon$</td>
<td>0.015</td>
<td>76</td>
</tr>
<tr>
<td>$K^- \rightarrow \pi^0 e^- \nu_\epsilon$</td>
<td>0.003</td>
<td>14</td>
</tr>
<tr>
<td>$\pi^- \rightarrow \mu^- \overline{\nu}<em>\mu \rightarrow e^- \overline{\nu}</em>\mu \nu_\epsilon$</td>
<td>0.001</td>
<td>4</td>
</tr>
<tr>
<td>$\Lambda^0 \rightarrow p e^- \overline{\nu}_\epsilon$</td>
<td>0.001</td>
<td>4</td>
</tr>
<tr>
<td><strong>total</strong></td>
<td>0.019</td>
<td>100</td>
</tr>
</tbody>
</table>
Figure 4.1: The calculated spectra for the four neutrino flavors in the wide band beam.
the total neutrino cross section. Next, the Monte Carlo chooses a nucleon for the neutrino to interact with assuming an equal number of protons and neutrons in the detector. The nucleon momentum is chosen according to the Fermi momentum distribution including the effects of Pauli blocking. The products of the interaction are chosen according to the relative cross sections of the possible processes. The following processes are considered:

1. Quasi-elastic scattering,

2. Charged-current and neutral-current single pion production,

3. Charge-current and neutral-current multiple pion production,


The four vectors of the interaction products are generated by straightforward dynamics for quasi-elastic elastic scattering. The product four vectors are generated using a model by Rein and Sehgal [40] for single pion production and a model from Kamiokande [41] for multiple pion production.

Two other processes are generated separately for $\nu_e$'s. The processes, neutrino-nucleus coherent $\pi^0$ production and neutrino-electron elastic scattering, are a small part of the total cross section but are important since the interaction products have a good chance of being misidentified as those of a $\nu_e$ interaction. They are generated separately to obtain a statistically larger sample of these interactions and later weighted appropriately. $\nu_e$'s, $\bar{\nu}_e$'s, and $\bar{\nu}_\mu$'s also undergo these processes but separate Monte Carlo is not generated since the flux of these neutrinos is so much lower. The product four vectors of neutrino-electron elastic scattering are generated using
straightforward dynamics and those of coherent $\pi^0$ production are generated by a
model by Rein and Schgal[42].

After determining the products of the interaction and their four vectors, those
products are propagated out of the nucleus. All interactions are assumed to take
place in an aluminum nucleus and the products are propagated independently and
not allowed to interfere with each other. The nuclear interactions are calculated by
NUCRIN[38]. After the four vectors of the interaction products are calculated for
when they leave the nucleus, the particles are propagated through the detector. The
interaction can take place anywhere in the detector.

The model detector in the Monte Carlo differs in many ways from the real E776
detector. The PDTs in the model are perfectly aligned which allows particles, particu-
larly muons, to travel through several planes without leaving a track. In the model,
particles do not interact with detector gas. The model also has an ideal magnetic
field in the toroids. An important difference is that the side walls of the PDTs are
ignored except for electrons with energy less than 1 $MeV$. The model does have the
same total mass as the real detector. In the model, all flash pulses have identical
shapes with the pulse area determined according to a Landau distribution taking into
account the particles path length through the PDT. In the real detector, pulse shape
and area depend on weather, gas quality, and distance from the wire, and real pulses
are much less uniform than those of the Monte Carlo.

The particles are propagated until they exit the detector, decay, or have negligible
energy. The particles undergo multiple scattering, magnetic bending, and dE/dx.
Electro-magnetic showers are handled completely by EGS4[43] and hadrons can in-
teract inelastically with nuclei as calculated by NUCRIN[38]. Each time the particle
cross a PDT, a pulse is recorded.

An attempt is made to simulate detector noise by superimposing the event Monte Carlo output on actual raw data frames. The output is then written out in the same format as the actual data. An additional record is written containing the known information such as the true identity and four vectors of the particles produced in the interactions.

To reduce statistical error, the number of Monte Carlo events generated is not the number of events we expect nor is the relative number of events of various flavors generated accurate. Also, in the $\nu_\mu$ Monte Carlo, events with a $\pi^0$ in the final state are generated five times more often than they would be based on cross section and given a weight one-fifth that of the rest of the $\nu_\mu$ Monte Carlo. The event Monte Carlo is weighted to give the number of charged-current events predicted by the beam Monte Carlo and the known cross section. A small correction to the weights is made based on the $\nu_\mu$ analysis to correct for the uncertainty in the number of protons on target.

The subsequent analysis is based on 500,000 $\nu_\mu$ Monte Carlo events, 17,000 $\nu_e$ Monte Carlo events, 49,000 $\bar{\nu}_e$ Monte Carlo events, 2,000 $\bar{\nu}_e$ Monte Carlo events, 9,000 coherent $\pi^0$ production Monte Carlo events, and 500 $\nu$-electron elastic scattering Monte Carlo events. Appropriately weighted, the Monte Carlo predicts in our detector 91,229 $\nu_\mu$ interactions (other than coherent $\pi^0$ production and $\nu$-e elastic scattering interactions), 1,858 $\bar{\nu}_\mu$ interactions, 658 $\nu_e$ interactions, 39 $\bar{\nu}_e$ interactions, 554 coherent $\pi^0$ production interactions, and 7 $\nu$-electron elastic scattering interactions. The predicted neutrino energy distribution of the various types of interacting neutrinos are shown in Figure 4.2, Figure 4.3, Figure 4.4, Figure 4.5, Figure 4.6, and Figure 4.7.
Figure 4.2: The Monte Carlo prediction for the neutrino energy spectrum of muon neutrino interactions (other than coherent $\pi^0$ production and $\nu_\mu$-electron elastic scattering interactions) in the E776 detector.
Figure 4.3: The Monte Carlo prediction for the neutrino energy spectrum of electron neutrino interactions in the E776 detector.
Figure 4.4: The Monte Carlo prediction for the neutrino energy spectrum of muon anti-neutrino interactions in the E776 detector.
Figure 4.5: The Monte Carlo prediction for the neutrino energy spectrum of electron anti-neutrino interactions in the E775 detector.
Figure 4.6: The Monte Carlo prediction for the neutrino energy spectrum of muon neutrino coherent $\pi^0$ production interactions in the E776 detector.
Figure 4.7: The Monte Carlo prediction for the neutrino energy spectrum of muon neutrino $\nu$-$e$ elastic scattering interactions in the E776 detector.
Chapter 5

Data Acquisition and Reduction

5.1 Data Acquisition

The Wide Band Run of E776 took place in the spring of 1986. Three different types of data were recorded. Beam data were recorded for each AGS pulse. This trigger was based on the AGS extraction signal (EXAU1). Absolute event times within the readout window are found using timing signals from the Čerenkov counters in the beam decay tunnel. $1.38 \times 10^6$ beam triggers were recorded.

400 $\text{msec}$ after each beam trigger, a free trigger was taken. The detector was read out exactly as for a beam trigger. These data are used to estimate the backgrounds which are not beam-related, for example, cosmic ray muons which enter the downstream end of the muon spectrometer and stop in the electromagnetic calorimeter. $1.36 \times 10^6$ free triggers were recorded.

Cosmic ray triggers were taken beginning 500 $\text{msec}$ before each beam trigger. For a cosmic ray trigger to be taken, certain criteria had to be met. Each plane of scintillator was divided into quadrants. If two of the phototubes connected to a quadrant fire, the quadrant was considered hit. If the same quadrant of seven of the ten scintillator planes was hit within a 4 $\mu\text{sec}$ gate, the entire event was written to
tape. 100 msec before the beam trigger, an abort signal stopped any writing. The cosmic rays were used to monitor detector performance and normalize the PDT pulse areas. About 0.6 cosmic ray triggers were taken for each beam trigger. \(8.1 \times 10^5\) cosmic ray triggers were recorded.

### 5.2 Data Reduction

Most of the beam triggers are empty frames. Many of the other beam triggers are events too complicated to be analyzed. In general, we are mainly interested in charged current quasi-elastic events because the neutrino energy can reconstructed using only the energy and angle of the lepton produced. To obtain a sample of desirable events of manageable size, the beam and free triggers are passed through several filter programs. The Monte Carlo events are also passed through the filters to monitor the filter programs' performance.

#### 5.2.1 Filters

The first filter program, known as Edit1, is designed to eliminate empty or nearly empty frames. At least ten PDTs must be hit for a frame to pass this filter. Also, at least three consecutive PDT planes must contain non-isolated hits. A hit is considered not isolated if there is another hit in its neighborhood. The neighborhood of a hit in plane N and wire M consists of wires M-2 through M+2 except wire M in plane N, wires M-4 through M+4 in planes N+2 and N-2, and wires M-6 through M+6 in planes N+4 and N-4. After Edit1, \(3.2 \times 10^5\) beam triggers (23\% of the original sample) and \(2.6 \times 10^4\) free triggers (2\% of the original sample) remain.

The Monte Carlo predicts 84\% of the \(\nu_\mu\) events (other than coherent \(\pi^0\) production
and $\nu$-e elastic scattering) in our detector will pass Edit1. 88% of the $\nu_e$ events, 84% of the $\bar{\nu}_e$ events, 85% of the $\bar{\nu}_\mu$ events, 71% of the coherent $\pi^0$ production events, and 91% for the $\nu$-e elastic scattering events are predicted to pass Edit1 by the Monte Carlo.

The total $\nu_e$ Edit1 acceptance is higher because the $\nu_e$ spectrum has a higher average energy and less low energy neutrino events which produce little or no activity in the detector. Many of the events which fail to pass this filter are neutral current events in which the products other than the neutrino had very low energy and the frame is nearly empty. Other events which fail to pass this filter are those where the interaction takes place near the edge of the detector and the products other than the neutrino exit the detector resulting in a nearly empty frame. If only quasi-elastic $\nu_e$ interactions which occur far from the edge of the detector and produce a lepton at a small angle relative to the beam direction are considered, all $\nu_e$ events pass this filter.

The second filter program, known as Edit3, is designed to eliminate frames which have some activity but no identifiable tracks. First, a fiducial volume is defined as wires 3 through 56 in planes 0 through 84. In order to be considered not isolated, a hit must be in the fiducial volume and there must be two other fiducially contained hits in its neighborhood. To pass this filter, there must be a non-isolated hit in four consecutive $x$ planes and in four consecutive $y$ planes. Secondly, a pattern recognition program attempts to find tracks in each frame. If a track is found, it must contain three hits in each view. The most upstream hit of the track in either view must be within the fiducial volume defined above. The most upstream hit of the track in the $x$ view must be within ten planes of the most upstream hit in the $y$ view. If no tracks are found by the pattern recognition, the event passes the filter unless more than 250
PDTs are hit, more than 80 PMTs are hit, or more than 10 muon spectrometer PDTs are hit. After Edit3, $7.8 \times 10^4$ beam triggers (6% of the original sample) and $6.4 \times 10^3$ free triggers (0.5% of the original sample) remain.

The Monte Carlo predicts 64% of the $\nu_\mu$ events (other than coherent $\pi^0$ production and $\nu$-e elastic scattering) in our detector will pass both filters. 68% of the $\nu_e$ events, 65% of the $\bar{\nu}_\mu$ events, 66% of the $\bar{\nu}_e$ events, 48% of the coherent $\pi^0$ production events, and 69% for the $\nu$-e elastic scattering events are predicted to pass both filters by the Monte Carlo.

Some of the events which do not pass Edit3 fail because they cause little activity in the detector while others fail because they cause too much activity. If only quasi-elastic $\nu_e$ interactions which occur far from the edge of the detector and produce a lepton at a small angle relative to the beam direction are considered, 97% of $\nu_e$ events pass both filters.

5.2.2 Pattern Recognition

Frames which passed Edit1 and Edit3 are measured by an automatic pattern recognition program called Autoscan. Autoscan begins by eliminating noise hits and keeping hits possibly caused by a neutrino event using the PDT times. The PDT times are histogrammed in 112 nsec bins and the bin with the most hits is found. Those PDTs which have times which fall between four bins (448 nsec) earlier and twenty-one bins ($\sim 2.4 \mu$sec) later than the bin with the most hits are categorized as event-related and all other PDT hits are treated as noise hits and not used in any later measurement or analysis.

If less than 35% of the hits in the frame are event-related or if there are less than
three event-related hits, Autoscan stops measuring the frame and it is considered unmeasurable. A frame is also considered unmeasurable if the peak of the hit time distribution is the first bin or after the fortieth bin.

After eliminating noise hits, Autoscan tries to find clusters of hits separately in each view using a Minimal Spanning Tree (MST)[44]. All remaining hits are connected two at a time such that the total length of the connecting lines is minimized. Any group of more than four PDT hits connected by lines of less than eight units in length is a cluster. The length of the connecting line between a hit in wire m of plane n to a hit in wire p of plane q is defined as $\sqrt{(p-m)^2 + (q-n)^2}$. Any frame where no clusters are found is considered unmeasurable. If more than one cluster is found in either view and the clusters contain hits from from the same planes, the frame is considered unmeasurable.

After finding clusters, Autoscan tries to find an event vertex in each view. Every cluster hit in the most upstream plane and the next four planes downstream in the same view is considered as a possible vertex. If there are no cluster hits in the next four planes downstream of the most upstream plane, cluster hits in the second most upstream plane in the view containing cluster hits are considered as a possible vertex. For each possible vertex, $N_1$ is defined as the number of cluster hits in the same view for which a line connecting the hit to the vertex candidate makes an angle between $0^\circ$ and $+5^\circ$ with the beam direction. In each view, a convention is established to distinguish positive from negative angles. $N_2, N_3, \ldots, N_7$ are defined similarly with $N_1$ being the number of cluster hits in the same view for which a line connecting the hit to the vertex candidate makes an angle between $+5 (i-1)$ degrees and $+5 i$ degrees with the beam direction. In each view, Autoscan chooses as the event vertex that vertex
candidate for which $\sum_{i=1}^{72} N_i^2$ is maximized. The frame is considered unmeasurable if the vertex found in the x view and the vertex found in the y view are more than five planes apart.

If a valid event vertex is found, Autoscan determines whether the frame contains a desirable single cluster event. To accomplish this, Autoscan first defines several variables in each view. $i_{\text{max}}$ is the value of $i$ for which $N_i$ is at a maximum. $N_{\text{max}}$ is the largest value of $N_i$. $N_{\text{peak}}$ is defined as $\sum_{i=i_{\text{max}}-3}^{i_{\text{max}}+3} N_i$ with $N_{69},...,N_{72}$ being equivalent to $N_{-3},...,N_0$ and $N_1,...,N_4$ being equivalent to $N_{73},...,N_{76}$. $N_{\text{PD}}$ is the total number of cluster hits and $N_{\text{out}}$ is $(N_{\text{PD}} - N_{\text{peak}})$. The event is considered a good single cluster event if, in both views, $N_{\text{out}}$ is less than 10, $N_{\text{peak}}$ is greater than 3, and $i_{\text{max}}$ is either less than 15 or greater than 58. In each view, those cluster hits counted as part of $N_{\text{peak}}$ are considered part of the "primary cluster" and those cluster hits counted as part of $N_{\text{out}}$ are considered "hits associated with the event."

If an event does not pass the single cluster event criteria, it is considered unmeasurable and not included in subsequent analysis, with two exceptions. The first exception is events for which, in the view(s) which does not meet the single cluster criteria, $N_{\text{max}}$ is less than 10, $i_{\text{max}}$ is between 16 and 67 (inclusive), and $N_{\text{PD}}$ is less than 21. These events are given a second chance. In the view(s) which does not meet the single cluster criteria, the most upstream hit in that view is taken as the vertex and the single cluster variables are recalculated and the single cluster criteria are applied again. The second exception is events for which both views failed the single cluster criteria and $N_{\text{max}}$ is greater than 9 in both views. These events are considered as possible M2 events: events containing a muon and one other single cluster in each view. The use of these events will be explained in the electron neutrino analysis.
chapter.

In determining if these possible M2 events are true M2 events, Autoscan first determines if the event contains a good muon by requiring, in both views, $i_{\text{max}}$ to be either less than 15 or greater than 68, $N_{\text{max}}$ to be greater than or equal to $0.85(N_{\text{max}}-1 + N_{\text{max}}+1)$, and the average normalized pulse area of the hits counted as part of $N_{\text{peak}}$ to be less than 350. If Autoscan determines a muon is present, a line is fitted to the hits counted as part of $N_{\text{peak}}$, in each view, and those hits closest to the line are considered part of the muon track.

Autoscan next determines if the cluster hits other than the muon track satisfy the single cluster criteria. Using the vertex found including the muon track, Autoscan recalculates the single cluster variables in each view, ignoring the muon track.

Now, the remaining hits pass the single cluster criteria if, in each view, $N_{\text{cut}}$ is less than 10, $N_{\text{peak}}$ is greater than 3, and $i_{\text{max}}$ is either less than 12 or greater than 60. The last requirement for an event to be an M2 events is that, in both views, $i_{\text{max}}$ including the muon track differ from $i_{\text{max}}$ ignoring the muon by at least 3. The difference between $i_{\text{max}}$ with the muon and $i_{\text{max}}$ without the muon is taken to be $|i_{\text{max \ with \ muon}} - i_{\text{max \ without \ muon}}|$, $|72 - i_{\text{max \ with \ muon}} + i_{\text{max \ without \ muon}}|$, or $|72 - i_{\text{max \ without \ muon}} + i_{\text{max \ with \ muon}}|$, whichever is smallest. In each view of an M2 event, those cluster hits counted as part of $N_{\text{peak}}$ when the muon is ignored are considered part of the “secondary cluster” and those cluster hits counted as part of $N_{\text{cut}}$ when the muon is ignored are considered “hits associated with the event.”

Autoscan classifies 41,831 beam data events and 1,871 free triggers as good single cluster events. Additionally, 1,306 beam data events are kept as an M2 sample. The Monte Carlo predicts that Autoscan will classify 35.553(39%) of the $\nu_\mu$ interactions
Table 5.1: The number of events remaining after successive data reduction stages. The $\nu_\mu$ prediction does not include coherent $\pi^0$ production or $\nu$-e elastic scattering.

(other than coherent $\pi^0$ production and $\nu$-e elastic scattering) in our detector as good single cluster events. According to the Monte Carlo, Autoscan will classify as good single cluster events 231(35%) of the $\nu_e$ interactions, 766(41%) of the $\bar{\nu}_\mu$ interactions, 17(44%) of the $\bar{\nu}_e$ interactions, 142(26%) of the coherent $\pi^0$ production interactions, and 4(59%) of the $\nu$-e elastic scattering interactions. The neutrino energy spectra of the events the Monte Carlo predicts will be classified as good single cluster events are shown in Figure 5.1, Figure 5.2, Figure 5.3, Figure 5.4, Figure 5.5, and Figure 5.6. The energy spectra of all the neutrinos the Monte Carlo predicts will interact are shown for comparison. Table 5.1 summarizes the data reduction.

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Figure 5.1: The Monte Carlo prediction for the neutrino energy spectrum of muon neutrino interactions (other than coherent $\pi^0$ production and $\nu$-$e$ elastic scattering) which are classified as good single cluster events by Autoscan (solid histogram) compared with the prediction for the neutrino energy spectrum of all such muon neutrino interactions in the E776 detector (dotted histogram).
Figure 5.2: The Monte Carlo prediction for the neutrino energy spectrum of electron neutrino interactions which are classified as good single cluster events by Autoscan (solid histogram) compared with the prediction for the neutrino energy spectrum of all electron neutrino interactions in the E776 detector (dotted histogram).
Figure 5.3: The Monte Carlo prediction for the neutrino energy spectrum of muon anti-neutrino interactions which are classified as good single cluster events by Autoscan (solid histogram) compared with the prediction for the neutrino energy spectrum of all muon anti-neutrino interactions in the E776 detector (dotted histogram).
Figure 5.4: The Monte Carlo prediction for the neutrino energy spectrum of electron anti-neutrino interactions which are classified as good single cluster events by Autoscan (solid histogram) compared with the prediction for the neutrino energy spectrum of all electron anti-neutrino interactions in the E776 detector (dotted histogram).
Figure 5.5: The Monte Carlo prediction for the neutrino energy spectrum of muon neutrino coherent $\pi^0$ production interactions which are classified as good single cluster events by Autoscan (solid histogram) compared with the prediction for the neutrino energy spectrum of all muon neutrino coherent $\pi^0$ production interactions in the E776 detector (dotted histogram).
Figure 5.6: The Monte Carlo prediction for the neutrino energy spectrum of muon neutrino $\nu$-e elastic scattering interactions which are classified as good single cluster events by Autoscan (solid histogram) compared with the prediction for the neutrino energy spectrum of all muon neutrino $\nu$-e elastic scattering interactions in the E776 detector (dotted histogram).
Chapter 6
Muon Neutrino Analysis

This chapter summarizes the muon neutrino analysis done by Greg Sullivan [35]. The purposes of the muon neutrino analysis are to find a normalization for the Monte Carlo and provide a check on both the beam and event Monte Carlo. The analysis uses only those data and Monte Carlo events classified as single cluster events by Autoscan. The analysis attempts to find quasi-elastic muon neutrino interactions because, in these events, the neutrino energy can be reconstructed from the angle and energy of the muon with Equation 1.22.

The energy of the muon is calculated in different ways for muons which stop in or exit the detector. If a muon stops in the detector, its energy is computed by $dE/dx$. The largest source of error in this method is the uncertainty in where the muon stopped in the absorber. For muons which stop in the calorimeter, the resolution is $\Delta E/E \approx 1.7\%$. For muons which stop in the toroids, $\Delta E/E \approx 9.89\%$. Muons which pass through the toroids and exit out the downstream end of the spectrometer are measured using magnetic bending. The uncertainty in this measurement is mainly due to the Coulomb multiple scattering the muon undergoes. The momentum of the muon as it enters the toroids is found with resolution $\Delta K/K \approx 22\%$, $K \equiv 1/p$. The
total energy of the muon is then found by $dE/dx$ with a resolution of $\Delta E/E \approx 15\%$. The angle of the muon is found by fitting a line to the hits classified as part of the primary cluster by Autoscan. The muon angle is found with resolution of better than $1^\circ$.

The muon analysis begins with a filter program to reduce the data and Monte Carlo samples to a more manageable size. The filter makes loose fiducial, containment, track length, and muon angle cuts. The vertex found by Autoscan, in either view, must not be in the first or last five planes of the detector or in the four exterior wires on either side of any plane. No hit classified as part of the primary cluster can be in the two exterior wires on either side of any plane. The polar angle between the primary cluster direction and the beam direction must be less than $45^\circ$. Lastly, if the muon track does not enter the toroids, the muon track length must be greater than 15 planes. After these cuts, 24,833 beam data events and 551 free triggers remain. The Monte Carlo predicts 20,614 $\nu_\mu$ and $\bar{\nu}_\mu$ events will pass these cuts.

The next cut of the muon analysis is that the toroid magnets must have been on during the run. This cut obviously has no effect on the Monte Carlo. Next, tighter fiducial and containment cuts are made. In both views, the vertex as found by Autoscan must not be in the 15 most downstream planes of the detector. The muon track is projected two planes downstream from the most downstream hit and must not pass through the two exterior wires on either side of any plane. The muon track and its projection must not pass through the hole in the toroids or outside the octagonal region covered by the toroids if it exits the downstream end of calorimeter. These cuts ensure the energy of events can be reconstructed with reasonable accuracy and help eliminate events caused by cosmic rays or beam related particles other than
neutrinos.

Next, cuts are applied to obtain a sample of events with only a single well-defined muon track characteristic of the desirable quasi-elastic muon neutrino interactions. The muon track must be at least 25 planes long. The polar angle of the muon track relative to the beam direction must be less than 37°. The reduced chi squared of the line fit to the muon track must be less than 5.0 in both views. In both views, the muon direction near the vertex must differ from the muon direction when it stops or enters the toroids by less than 5°. The muon track must cause two or more PDT hits in the same plane in no more than $\frac{1}{5}$ of the planes it transverses. If the muon track penetrates the toroids, the muon must not be defocused by the magnets. In each view, there can be no more than 5 PDT hits classified by Autoscan as hits associated with the event. Lastly the reconstructed neutrino energy must be greater than zero.

After these cuts, 5,581 beam data events and 2 free triggers remain. The two free triggers indicate that 2 of the 5,581 beam events were the result of something other than beam neutrinos, e.g., cosmic rays. The Monte Carlo predicts 5,592 events should pass these cuts: 5,550 $\nu_\mu$ events and 42 $\overline{\nu}_\mu$ events. The Monte Carlo indicates that 53% of the events in the final sample are from quasi-elastic interactions, the rest are mostly from single pion and multiple pion production. Table 6.1 shows the number of events remaining after each successive cut. The $\nu_\mu$ and $\overline{\nu}_\mu$ overall acceptances as a function of energy are shown in Figure 6.1 and Figure 6.2. The reconstructed neutrino energy spectra of the data and the Monte Carlo prediction are compared in Figure 6.3. The reconstructed $\cos\theta_\mu$ distribution for the data and Monte Carlo prediction are compared in Figure 6.4. The good agreement between the total number of events in the data and Monte Carlo prediction is not surprising
Table 6.1: The number of $\nu_\mu$ events remaining after successive cuts.

since this analysis set the Monte Carlo normalization. However, the good agreement between the reconstructed neutrino energy spectra and $\cos \theta_\mu$ distribution gives us confidence in our Monte Carlo.
Figure 6.1: The overall acceptance of $\nu_\mu$ Monte Carlo events vs. incident neutrino energy. (a) Acceptance for all $\nu_\mu$ events. (b) Acceptance for quasi-elastic $\nu_\mu$ events.
Figure 6.2: The overall acceptance for all $\bar{\nu}_e$ Monte Carlo events vs. incident neutrino energy.
Figure 6.3: Comparison of the reconstructed $\nu_\mu$ spectra for the data (points) and the Monte Carlo prediction (histogram).
Figure 6.4: Comparison of the reconstructed $\cos \theta_\mu$ distribution of the data(points) and the Monte Carlo prediction(histogram).
Chapter 7

Electron Neutrino Analysis

The purpose of the electron neutrino analysis is to search for quasi-elastic electron neutrino events in the data. An excess of such events compared to the Monte Carlo prediction of the events we should find due to beam contamination and misidentified muon neutrino events would be evidence for neutrino oscillations. The analysis again searches for quasi-elastic interactions because the neutrino energy can be easily reconstructed.

The energy of the electron is computed using the total normalized pulse area of the hits classified as part of the primary cluster by Autoscan. The pulse areas are normalized [25] using the cosmic ray data to take into account variations from wire to wire and over time due to weather, gas quality, and electronics. Based on data from the A2 test run (see Appendix A), the electron pulse area varies linearly with energy as shown in Figure 7.1. The electron energy resolution is about $20\%\sqrt{E}$. The electron angle is found by fitting a line to the primary cluster hits weighted by their normalized pulse areas. The electron angular resolution is about $2^\circ$.

The electron analysis begins with those data and Monte Carlo events which Autoscan classifies as single cluster events. Those data events from runs when the toroids
Figure 7.1: Normalized pulse area as a function of energy for A2 test electrons.
were off are again cut.

The first set of cuts consists of a fiducial cut, an angle cut, a containment cut, and a vertex activity cut. The vertex found by Autoscan must be between planes 15 and 84 and wires 4 and 58 (inclusive). The polar angle of the primary cluster must be less than 37°. No hit considered part of the primary cluster may be in the toroids, plane 0 or 89 of the calorimeter, or wire 0 or 63 of any calorimeter plane. Lastly, there can be no more than 5 hits classified as hits associated with the event by Autoscan in each view. The data and Monte Carlo events which pass these cuts make up the FACQ sample. There are 3,358 beam data events and 7 free triggers in the FACQ sample. The Monte Carlo predicts 3,223 events in the FACQ sample. Of the 3,223 events the Monte Carlo predicts, 3,154 events are “π° events” and 69 are “electron events”. Those events from \( \nu_\mu \) (other than \( \nu-e \) elastic scattering) or \( \bar{\nu}_\mu \) interactions are referred to as \( \pi^0 \) events since if these events are mistaken for \( \nu_e \) interactions, it is typically due to a \( \pi^0 \) being mistaken for an electron. Events from \( \nu_e, \bar{\nu}_e, \) or \( \nu-e \) elastic scattering interactions are referred to as electron events because the final state includes an electron. The energy spectra of the data FACQ sample and the Monte Carlo prediction are shown in Figure 7.2. The FACQ acceptance of \( \nu_e \) events based on the Monte Carlo is shown in Figure 7.3.

The next set of four cuts is designed to keep only events in which the primary cluster is a single electro-magnetic shower. In our detector, electro-magnetic showers are characterized by a dense, well-collimated core and a discontinuous hit pattern due to the exchange of energy between photons and electrons. Those events which pass these cuts make up the shower sample.

The first cut requires that the gapiness of the primary cluster be greater than
Figure 7.2: Reconstructed neutrino energy spectra for the FACQ sample comparing the data sample (with error bars) with the Monte Carlo prediction (solid histogram).
Figure 7.3: FACQ acceptance from Monte Carlo as a function of neutrino energy for all $\nu_e$ events.
0.4. The gappiness of a cluster, $G$, is defined as

$$G = \frac{1}{D} \sum_{p=1}^{\infty} |N_p - N_{p-1}|$$ (7.1)

where $N_p$ is the number of PDT hits in plane $p$ classified as part of the primary cluster by Autoscan and $D$ is the number of planes between the most upstream hit of the primary cluster and the most downstream hit of the primary cluster (inclusive). This cut attempts to quantify the fact that showers are discontinuous or "skippy".

The second cut requires that the pulse area per plane of the primary cluster be greater than 300. The pulse area per plane, $P$, is defined as

$$P = \frac{\cos \theta_e}{D} \sum_{i=1}^{N} (PA)_i$$ (7.2)

where $D$ is as above, $\theta_e$ is the polar angle of the primary cluster calculated assuming it is an electron, $(PA)_i$ is the normalized pulse area of PDT hit $i$, and $N$ is the total number of PDT hits in the primary cluster. This cut attempts to quantify the fact that showers are dense. In an electro-magnetic shower, several particles pass through each plane and deposit energy. The normalized pulse area of a single minimum ionizing particle is about 100.

The third cut requires that the length of the primary cluster be within $1.5\sigma$ of the value expected based on test beam studies. The length is measured from the vertex along the shower axis determined by pulse area weighted fit to the primary cluster hits. The length of the cluster is the length required to contain 80% of the total pulse area of the cluster. This length is compared to the value expected for a test data electron with the same total pulse area.

The fourth cut requires that the cluster be well collimated. In each view, the
eigenvalues, $\lambda_1$ and $\lambda_2$, are found for the matrix $M$ defined as

$$M = \begin{bmatrix} I_{11} & I_{12} \\ I_{21} & I_{22} \end{bmatrix}$$

(7.3)

where $I_{ij} = \sum_{\alpha=1}^{N_{\text{view}}} \delta_{ij} \bar{p}_a^2 - \bar{p}_a \bar{p}_\alpha$ where $\bar{p}_\alpha = (PA)_\alpha \times \bar{r}_\alpha$ for hit $\alpha$ where $\bar{r}_\alpha$ is the vector from the Autoscan event vertex to hit $\alpha$ and $N_{\text{view}}$ is the total number of primary cluster PDT hits in the view. The ratio of the eigenvalues, $R$, is the ratio of the smaller eigenvalue to the larger eigenvalue. That is,

$$R = \frac{\lambda_1}{\lambda_2}$$

(7.4)

with $\lambda_1 < \lambda_2$. There are two values of $R$, one for each view. $R$ is required to be less than 0.004 in at least one view.

There are 111 beam data events and no free triggers in the shower sample. The Monte Carlo predicts 94 events in the shower sample: 33 electron events and 61 $\pi^0$ events. The energy spectra of the data shower sample and Monte Carlo prediction are shown in Figure 7.4. The shower acceptance of $\nu_e$ events based on Monte Carlo is shown in Figure 7.5. The shower acceptance of A2 test beam electrons is shown in Figure 7.6. Table 7.1 summarizes the effect of each shower cut on the data and Monte Carlo prediction. The distributions of the cut variables are shown in Figure 7.7, Figure 7.8, Figure 7.9, and Figure 7.10 and the effect of each individual cut on the data and Monte Carlo is shown in Figure 7.11, Figure 7.12, Figure 7.13, and Figure 7.14.

The events in the shower sample are next passed through a set of four cuts designed to keep electron events and eliminate $\pi^0$ events. The shower profile of a $\pi^0$ differs from that of an electron of the same energy. In general, $\pi^0$s are wider and more asymmetric than electrons. Those events which pass these cuts make up the final electron sample. The first cut requires the ratio of eigenvalues used in the shower
Figure 7.4: Reconstructed neutrino energy spectra for the shower sample. (a) Comparison of the data sample (with error bars) with the Monte Carlo prediction (solid histogram). (b) Contributions to the Monte Carlo prediction from $\pi^0$ events (solid histogram) and $e^-$ events (dotted histogram).
Figure 7.5: Shower acceptance from Monte Carlo as a function of neutrino energy for (a) all $\nu_e$ events, (b) quasi-elastic $\nu_e$ events.
Figure 7.6: Shower acceptance for A2 test electrons as a function of beam energy for (a) the beam along the detector axis, (b) the beam at 30° relative to the detector axis.
Figure 7.7: (a) The gappiness distribution of the data (with error bars) and the Monte Carlo prediction (histogram) in the FACQ sample. (b) The gappiness distribution of the data (with error bars) and the Monte Carlo prediction (histogram) in the FACQ sample which pass the other three shower cuts. (c) The gappiness distribution of the Monte Carlo prediction of $\pi^0$ events in the FACQ sample. (d) The gappiness distribution of the Monte Carlo prediction of $\pi^0$ events in the FACQ sample which pass the other three shower cuts. (e) The gappiness distribution of the Monte Carlo prediction of electron events in the FACQ sample. (f) The gappiness distribution of the Monte Carlo prediction of electron events in the FACQ sample which pass the other three shower cuts.
Figure 7.8: (a) The pulse area per plane distribution of the data (with error bars) and the Monte Carlo prediction (histogram) in the FACQ sample. (b) The pulse area per plane distribution of the data (with error bars) and the Monte Carlo prediction (histogram) in the FACQ sample which pass the other three shower cuts. (c) The pulse area per plane distribution of the Monte Carlo prediction of $\pi^0$ events in the FACQ sample. (d) The pulse area per plane distribution of the Monte Carlo prediction of $\pi^0$ events in the FACQ sample which pass the other three shower cuts. (e) The pulse area per plane distribution of the Monte Carlo prediction of electron events in the FACQ sample. (f) The pulse area per plane distribution of the Monte Carlo prediction of electron events in the FACQ sample which pass the other three shower cuts.
Figure 7.9: (a) The length difference (in sigmas) distribution of the data (with error bars) and the Monte Carlo prediction (histogram) in the FACQ sample. (b) The length difference (in sigmas) distribution of the data (with error bars) and the Monte Carlo prediction (histogram) in the FACQ sample which pass the other three shower cuts. (c) The length difference (in sigmas) distribution of the Monte Carlo prediction of π⁰ events in the FACQ sample. (d) The length difference (in sigmas) distribution of the Monte Carlo prediction of π⁰ events in the FACQ sample which pass the other three shower cuts. (e) The length difference (in sigmas) distribution of the Monte Carlo prediction of electron events in the FACQ sample. (f) The length difference (in sigmas) distribution of the Monte Carlo prediction of electron events in the FACQ sample which pass the other three shower cuts.
Figure 7.10: (a) The ratio of eigenvalues distribution of the data (with error bars) and the Monte Carlo prediction (histogram) in the FACQ sample. (b) The ratio of eigenvalues distribution of the data (with error bars) and the Monte Carlo prediction (histogram) in the FACQ sample which pass the other three shower cuts. (c) The ratio of eigenvalues distribution of the Monte Carlo prediction of π⁰ events in the FACQ sample. (d) The ratio of eigenvalues distribution of the Monte Carlo prediction of π⁰ events in the FACQ sample which pass the other three shower cuts. (e) The ratio of eigenvalues distribution of the Monte Carlo prediction of electron events in the FACQ sample. (f) The ratio of eigenvalues distribution of the Monte Carlo prediction of electron events in the FACQ sample which pass the other three shower cuts.
Figure 7.11: (a) Reconstructed neutrino energy spectra of events which fail the gap-piness cut for the data (with error bars) and Monte Carlo prediction (histogram) in the FACQ sample. (b) Reconstructed neutrino energy spectra of events which fail the gap-piness cut but pass the other three shower cuts for the data (with error bars) and Monte Carlo prediction (histogram) in the FACQ sample. (c) Reconstructed neutrino energy spectra of events which fail the gap-piness cut for the Monte Carlo prediction of $\pi^0$ events in the FACQ sample. (d) Reconstructed neutrino energy spectra of events which fail the gap-piness cut but pass the other three shower cuts for the Monte Carlo prediction of $\pi^0$ events in the FACQ sample. (e) Reconstructed neutrino energy spectra of events which fail the gap-piness cut for the Monte Carlo prediction of electron events in the FACQ sample. (f) Reconstructed neutrino energy spectra of events which fail the gap-piness cut but pass the other three shower cuts for the Monte Carlo prediction of electron events in the FACQ sample.
Figure 7.12: (a) Reconstructed neutrino energy spectra of events which fail the pulse area per plane cut for the data (with error bars) and Monte Carlo prediction (histogram) in the FACQ sample. (b) Reconstructed neutrino energy spectra of events which fail the pulse area per plane cut but pass the other three shower cuts for the data (with error bars) and Monte Carlo prediction (histogram) in the FACQ sample. (c) Reconstructed neutrino energy spectra of events which fail the pulse area per plane cut for the Monte Carlo prediction of $\pi^0$ events in the FACQ sample. (d) Reconstructed neutrino energy spectra of events which fail the pulse area per plane cut but pass the other three shower cuts for the Monte Carlo prediction of $\pi^0$ events in the FACQ sample. (e) Reconstructed neutrino energy spectra of events which fail the pulse area per plane cut for the Monte Carlo prediction of electron events in the FACQ sample. (f) Reconstructed neutrino energy spectra of events which fail the pulse area per plane cut but pass the other three shower cuts for the Monte Carlo prediction of electron events in the FACQ sample.
Figure 7.13: (a) Reconstructed neutrino energy spectra of the events which fail the length cut for the data (with error bars) and Monte Carlo prediction (histogram) in the FACQ sample. (b) Reconstructed neutrino energy spectra of events which fail the length cut but pass the other three shower cuts for the data (with error bars) and Monte Carlo prediction (histogram) in the FACQ sample. (c) Reconstructed neutrino energy spectra of events which fail the length cut for the Monte Carlo prediction of π⁰ events in the FACQ sample. (d) Reconstructed neutrino energy spectra of events which fail the length cut but pass the other three shower cuts for the Monte Carlo prediction of π⁰ events in the FACQ sample. (e) Reconstructed neutrino energy spectra of events which fail the length cut for the Monte Carlo prediction of electron events in the FACQ sample. (f) Reconstructed neutrino energy spectra of events which fail the length cut but pass the other three shower cuts for the Monte Carlo prediction of electron events in the FACQ sample.
Figure 7.14: (a) Reconstructed neutrino energy spectra of events which fail the ratio of eigenvalues cut for the data (with error bars) and Monte Carlo prediction (histogram) in the FACQ sample. (b) Reconstructed neutrino energy spectra of events which fail the ratio of eigenvalues cut but pass the other three shower cuts for the data (with error bars) and Monte Carlo prediction (histogram) in the FACQ sample. (c) Reconstructed neutrino energy spectra of events which fail the ratio of eigenvalues cut for the Monte Carlo prediction of $\pi^0$ events in the FACQ sample. (d) Reconstructed neutrino energy spectra of events which fail the ratio of eigenvalues cut but pass the other three shower cuts for the Monte Carlo prediction of $\pi^0$ events in the FACQ sample. (e) Reconstructed neutrino energy spectra of events which fail the ratio of eigenvalues cut for the Monte Carlo prediction of electron events in the FACQ sample. (f) Reconstructed neutrino energy spectra of events which fail the ratio of eigenvalues cut but pass the other three shower cuts for the Monte Carlo prediction of electron events in the FACQ sample.
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Table 7.1: The number of FACQ sample events which fail each shower cut inclusively and exclusively. The $\nu_\mu$ prediction does not include coherent $\pi^0$ production or $\nu-e$ elastic scattering.

cuts must be less than 0.0025 in at least one view.

Each PDT has 8 neighboring PDTs. For wire $w$ in plane $p$, the neighbors are wires $w-1$ and $w+1$ in planes $p,p-2$, and $p+2$, and wire $w$ in planes $p-2$ and $p+2$. If neighboring PDTs have primary cluster hits, those hits are considered to be each other's neighbor. The second cut requires the primary cluster hits have an average of at least 2.65 neighbors.

The third cut requires the shower profile to be similar to that of an electron. The cut simply requires the total pulse area in the upstream half of the shower to be less than 1.1 times the total pulse area in the downstream half of the detector. The shower is divided along the beam direction based on the plane of the most upstream and most downstream hit in the primary cluster. If the shower spans an even number of planes, the division is simple and if it spans an odd number of planes, the midplane is included in the downstream half.
CHAPTER 7. ELECTRON NEUTRINO ANALYSIS

After the previous three cuts, several events remain which appear to be \( \pi^0 \)'s to the eye. These events exhibit the "V" shape typical of \( \pi^0 \) events because the \( \pi^0 \) decays to two photons. The last cut attempts to eliminate these events. Events are cut if, in either view, there are three consecutive planes which have a "space" in them. In the x view (y view), a space is a PDT with no primary cluster hit for which there are PDTs with primary cluster hits both to the right and left (above and below). The Monte Carlo predicts much fewer events will fail this cut than are found in the data. These events are clearly not quasi-elastic \( \nu_e \) events but it is unclear why the Monte Carlo predicts so many fewer.

There are 37 beam data events and no free triggers in the final electron sample. The Monte Carlo predicts 36.2 events in the final electron sample: 22.5 electron events and 13.7 \( \pi^0 \) events. The energy spectra of the data final electron sample and the Monte Carlo prediction are shown in Figure 7.15. The final electron acceptance of \( \nu_e \) events based on the Monte Carlo in Figure 7.16. The final electron acceptance of A2 test beam electrons is shown in Figure 7.17. Table 7.2 summarizes the effect of each electron cut on the data and Monte Carlo prediction. The distributions of the cut variables are shown in Figure 7.18, Figure 7.19, and Figure 7.20 and the effect of each individual cut on the data and Monte Carlo are shown in Figure 7.21, Figure 7.22, Figure 7.23, and Figure 7.24.

The reconstructed \( \cos \theta_e \) distribution for the data and Monte Carlo prediction for the final electron sample are compared in Figure 7.25. The electron neutrino analysis is summarized in Table 7.3. The actual neutrino energy and reconstructed neutrino energy are often quite different. The actual neutrino energy and reconstructed neutrino energy spectra are compared for electron Monte Carlo events and \( \pi^0 \) Monte
Figure 7.15: Reconstructed neutrino energy spectra for the final electron sample. (a) Comparison of the data sample (with error bars) with the Monte Carlo prediction (solid histogram). (b) Contributions to the Monte Carlo prediction from $\pi^0$ events (solid histogram) and $e^-$ events (dotted histogram).
Figure 7.16: Final electron acceptance from Monte Carlo as a function of neutrino energy for (a) all $\nu_e$ events, (b) quasi-elastic $\nu_e$ events.
Figure 7.17: Final electron acceptance for A2 test electrons as a function of beam energy for (a) the beam along the detector axis, (b) the beam at 30° relative to the detector axis.
Figure 7.18: (a) The average neighbors distribution of the data (with error bars) and the Monte Carlo prediction (histogram) in the shower sample. (b) The average neighbors distribution of the data (with error bars) and the Monte Carlo prediction (histogram) in the shower sample which pass the other three electron cuts. (c) The average neighbors distribution of the Monte Carlo prediction of π^0 events in the shower sample. (d) The average neighbors distribution of the Monte Carlo prediction of π^0 events in the shower sample which pass the other three electron cuts. (e) The average neighbors distribution of the Monte Carlo prediction of electron events in the shower sample. (f) The average neighbors distribution of the Monte Carlo prediction of electron events in the shower sample which pass the other three electron cuts.
Figure 7.19: (a) The ratio of downstream to upstream pulse area distribution of the data (with error bars) and the Monte Carlo prediction (histogram) in the shower sample. (b) The ratio of downstream to upstream pulse area distribution of the data (with error bars) and the Monte Carlo prediction (histogram) in the shower sample which pass the other three electron cuts. (c) The ratio of downstream to upstream pulse area distribution of the Monte Carlo prediction of $\pi^0$ events in the shower sample. (d) The ratio of downstream to upstream pulse area distribution of the Monte Carlo prediction of $\pi^0$ events in the shower sample which pass the other three electron cuts. (e) The ratio of downstream to upstream pulse area distribution of the Monte Carlo prediction of electron events in the shower sample. (f) The ratio of downstream to upstream pulse area distribution of the Monte Carlo prediction of electron events in the shower sample which pass the other three electron cuts.
Figure 7.20: (a) The ratio of eigenvalues distribution of the data (with error bars) and the Monte Carlo prediction (histogram) in the shower sample. (b) The ratio of eigenvalues distribution of the data (with error bars) and the Monte Carlo prediction (histogram) in the shower sample which pass the other three electron cuts. (c) The ratio of eigenvalues distribution of the Monte Carlo prediction of $\tau^0$ events in the shower sample. (d) The ratio of eigenvalues distribution of the Monte Carlo prediction of $\tau^0$ events in the shower sample which pass the other three electron cuts. (e) The ratio of eigenvalues distribution of the Monte Carlo prediction of electron events in the shower sample. (f) The ratio of eigenvalues distribution of the Monte Carlo prediction of electron events in the shower sample which pass the other three electron cuts.
Figure 7.21: (a) Reconstructed neutrino energy spectra of events which fail the average neighbors cut for the data (with error bars) and Monte Carlo prediction (histogram) in the shower sample. (b) Reconstructed neutrino energy spectra of events which fail the average neighbors cut but pass the other three electron cuts for the data (with error bars) and Monte Carlo prediction (histogram) in the shower sample. (c) Reconstructed neutrino energy spectra of events which fail the average neighbors cut for the Monte Carlo prediction of $\pi^0$ events in the shower sample. (d) Reconstructed neutrino energy spectra of events which fail the average neighbors cut but pass the other three electron cuts for the Monte Carlo prediction of $\pi^0$ events in the shower sample. (e) Reconstructed neutrino energy spectra of events which fail the average neighbors cut for the Monte Carlo prediction of electron events in the shower sample. (f) Reconstructed neutrino energy spectra of events which fail the average neighbors cut but pass the other three electron cuts for the Monte Carlo prediction of electron events in the shower sample.
Figure 7.22: (a) Reconstructed neutrino energy spectra of events which fail the ratio of downstream to upstream pulse area cut for the data (with error bars) and Monte Carlo prediction (histogram) in the shower sample. (b) Reconstructed neutrino energy spectra of events which fail the ratio of downstream to upstream pulse area cut but pass the other three electron cuts for the data (with error bars) and Monte Carlo prediction (histogram) in the shower sample. (c) Reconstructed neutrino energy spectra of events which fail the ratio of downstream to upstream pulse area cut for the Monte Carlo prediction of $\pi^0$ events in the shower sample. (d) Reconstructed neutrino energy spectra of events which fail the ratio of downstream to upstream pulse area cut but pass the other three electron cuts for the Monte Carlo prediction of $\pi^0$ events in the shower sample. (e) Reconstructed neutrino energy spectra of events which fail the ratio of downstream to upstream pulse area cut for the Monte Carlo prediction of electron events in the shower sample. (f) Reconstructed neutrino energy spectra of events which fail the ratio of downstream to upstream pulse area cut but pass the other three electron cuts for the Monte Carlo prediction of electron events in the shower sample.
Figure 7.23: (a) Reconstructed neutrino energy spectra of events which fail the space cut for the data (with error bars) and Monte Carlo prediction (histogram) in the shower sample. (b) Reconstructed neutrino energy spectra of events which fail the space cut but pass the other three electron cuts for the data (with error bars) and Monte Carlo prediction (histogram) in the shower sample. (c) Reconstructed neutrino energy spectra of events which fail the space cut for the Monte Carlo prediction of π^0 events in the shower sample. (d) Reconstructed neutrino energy spectra of events which fail the space cut but pass the other three electron cuts for the Monte Carlo prediction of π^0 events in the shower sample. (e) Reconstructed neutrino energy spectra of events which fail the space cut for the Monte Carlo prediction of electron events in the shower sample. (f) Reconstructed neutrino energy spectra of events which fail the space cut but pass the other three electron cuts for the Monte Carlo prediction of electron events in the shower sample.
Figure 7.24: (a) Reconstructed neutrino energy spectra of events which fail the ratio of eigenvalues cut for the data (with error bars) and Monte Carlo prediction (histogram) in the shower sample. (b) Reconstructed neutrino energy spectra of events which fail the ratio of eigenvalues cut but pass the other three electron cuts for the data (with error bars) and Monte Carlo prediction (histogram) in the shower sample. (c) Reconstructed neutrino energy spectra of events which fail the ratio of eigenvalues cut for the Monte Carlo prediction of $\pi^0$ events in the shower sample. (d) Reconstructed neutrino energy spectra of events which fail the ratio of eigenvalues cut but pass the other three electron cuts for the Monte Carlo prediction of $\pi^0$ events in the shower sample. (e) Reconstructed neutrino energy spectra of events which fail the ratio of eigenvalues cut for the Monte Carlo prediction of electron events in the shower sample. (f) Reconstructed neutrino energy spectra of events which fail the ratio of eigenvalues cut but pass the other three electron cuts for the Monte Carlo prediction of electron events in the shower sample.
<table>
<thead>
<tr>
<th>Category</th>
<th>Data</th>
<th>Monte Carlo Prediction</th>
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<tr>
<td></td>
<td>Beam</td>
<td>Free</td>
</tr>
<tr>
<td>Shower sample</td>
<td>110</td>
<td>1</td>
</tr>
<tr>
<td>Fail neighbors (inc)</td>
<td>33</td>
<td>1</td>
</tr>
<tr>
<td>Fail neighbors (exc)</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>Fail profile (inc)</td>
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<td>0</td>
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<tr>
<td>Fail space (exc)</td>
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<tr>
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<tr>
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<tr>
<td>Electron sample</td>
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Table 7.2: The number of shower sample events which fail each electron cut inclusively and exclusively. The $\nu_\mu$ prediction does not include coherent $\pi^0$ production or $\nu-e$ elastic scattering.

Carlo events in the final electron sample in Figure 7.26. The Monte Carlo prediction for the contributions of the various types of neutrino interactions to the final electron sample is shown in Table 7.4.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Data</th>
<th>MC</th>
<th>$\nu_\mu$</th>
<th>$\overline{\nu}_\mu$</th>
<th>$\nu_e$</th>
<th>$\overline{\nu}_e$</th>
<th>Coh $\pi^0$</th>
<th>$\nu-e$</th>
</tr>
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<tbody>
<tr>
<td>Fid, Con, Act</td>
<td>3365</td>
<td>3223</td>
<td>60.8</td>
<td>5.6</td>
<td>2843</td>
<td>31.2</td>
<td>75.6</td>
<td>2.3</td>
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<tr>
<td>Shower</td>
<td>111</td>
<td>94</td>
<td>28.9</td>
<td>3.2</td>
<td>51.4</td>
<td>0.6</td>
<td>8.2</td>
<td>0.9</td>
</tr>
<tr>
<td>Electron</td>
<td>37</td>
<td>36.2</td>
<td>19.6</td>
<td>2.1</td>
<td>9.7</td>
<td>0.2</td>
<td>3.8</td>
<td>0.8</td>
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</tbody>
</table>

Table 7.3: A summary of the electron analysis. The first row is after vertex fiducial, track containment and vertex activity cuts, the second and third rows are after the electromagnetic shower and electron cuts respectively.

The Monte Carlo predicts 36.2 events in the final electron sample. 22.5 of these events actually contain an electron in the final state: 19.6 from $\nu_e$ interactions, 2.1 from $\overline{\nu}_e$ interactions, and 0.8 from $\nu-e$ elastic scattering. The systematic error on these predictions is estimated as 10.5% from uncertainty in the beam calculation.
Figure 7.25: Comparison of the reconstructed $\cos \theta_c$ distributions for the data (with error bars) and the Monte Carlo prediction (histogram).
Figure 7.26: (a) The true neutrino energy spectrum (solid histogram) and reconstructed neutrino energy spectrum (dotted histogram) for Monte Carlo electron events in the final electron sample. (b) The true neutrino energy spectrum (solid histogram) and reconstructed neutrino energy spectrum (dotted histogram) for Monte Carlo $\pi^0$ events in the final electron sample.
<table>
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<tr>
<th>Type of Neutrino Interaction</th>
<th>Predicted Contribution (Number of events)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_e n \rightarrow e^- p$</td>
<td>11.039</td>
</tr>
<tr>
<td>$\nu_\mu N \rightarrow \nu_\mu N' \pi \pi ...$</td>
<td>5.566</td>
</tr>
<tr>
<td>$\nu_\mu N \rightarrow \mu_\mu N \pi^0$</td>
<td>3.818</td>
</tr>
<tr>
<td>$\nu_\mu p \rightarrow e^- \mu \pi^+$</td>
<td>3.331</td>
</tr>
<tr>
<td>$\nu_\mu N \rightarrow e^- N' \pi \pi ...$</td>
<td>2.595</td>
</tr>
<tr>
<td>$\nu_\mu N \rightarrow \mu^- N' \pi \pi ...$</td>
<td>2.269</td>
</tr>
<tr>
<td>$\nu_e n \rightarrow e^- \pi^+$</td>
<td>1.666</td>
</tr>
<tr>
<td>$\bar{\nu}_e p \rightarrow e^+ n$</td>
<td>1.220</td>
</tr>
<tr>
<td>$\nu_e n \rightarrow e^- \pi^+$</td>
<td>0.930</td>
</tr>
<tr>
<td>$\nu_\mu p \rightarrow \nu_\mu p \pi^0$</td>
<td>0.811</td>
</tr>
<tr>
<td>$\nu_\mu e^- \rightarrow $ $\nu_\mu e^-$</td>
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<tr>
<td>$\bar{\nu}_e n \rightarrow e^+ n \pi^-$</td>
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</tr>
<tr>
<td>$\nu_\mu n \rightarrow \nu_\mu n \pi^0$</td>
<td>0.378</td>
</tr>
<tr>
<td>$\nu_\mu n \rightarrow \mu^- \pi^0$</td>
<td>0.270</td>
</tr>
<tr>
<td>$\nu_\mu p \rightarrow \nu_\mu n \pi^+$</td>
<td>0.270</td>
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<tr>
<td>$\bar{\nu}_e p \rightarrow e^+ p \pi^-$</td>
<td>0.194</td>
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<tr>
<td>$\bar{\nu}<em>\mu N \rightarrow \bar{\nu}</em>\mu N' \pi \pi ...$</td>
<td>0.193</td>
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<tr>
<td>$\bar{\nu}_e N \rightarrow e^+ N' \pi \pi ...$</td>
<td>0.136</td>
</tr>
<tr>
<td>$\bar{\nu}_e p \rightarrow e^+ n \pi^0$</td>
<td>0.116</td>
</tr>
<tr>
<td>$\nu_\mu N \rightarrow \nu_\mu N' \pi \pi ...$</td>
<td>0.077</td>
</tr>
<tr>
<td>$\nu_\mu p \rightarrow \nu_\mu p$</td>
<td>0.054</td>
</tr>
<tr>
<td>$\bar{\nu}<em>\mu n \rightarrow \bar{\nu}</em>\mu n \pi^0$</td>
<td>0.023</td>
</tr>
<tr>
<td>$\bar{\nu}<em>\mu p \rightarrow \bar{\nu}</em>\mu p \pi^0$</td>
<td>0.011</td>
</tr>
</tbody>
</table>

Table 7.4: The Monte Carlo prediction for the contributions of the various types of neutrino interactions to the final electron sample.
(10%) and systematic error in the flux normalization (3.5%).

The Monte Carlo predicts that 13.7 events in the final electron sample contain no electron in the final state. Generally, a $\pi^0$ was mistaken for an electron. 9.7 of these events are from $\nu_\mu$ interactions (other than $\nu_e$ elastic scattering and coherent $\pi^0$ production), 0.2 from $\bar{\nu}_\mu$ interactions, and 3.8 from coherent $\pi^0$ production. The systematic error on these predictions is dominated by the large unknown uncertainty in some of the cross sections in the Monte Carlo. To estimate this systematic error, the number of $\pi^0$ events in the final electron sample is calculated in two additional ways.

The first method calculates the number of $\pi^0$ events in the final electron sample, $N_{\pi^0 \rightarrow "e"}$, as

$$N_{\pi^0 \rightarrow "e"} = N_{\pi^0 \rightarrow "\pi^0"} \times \frac{P(\pi^0 \rightarrow "e")}{P(\pi^0 \rightarrow "\pi^0")}$$  \hspace{1cm} (7.5)

where $N_{\pi^0 \rightarrow "\pi^0"}$ is the number of $\pi^0$ events in the data shower sample which do not make the final electron sample, $P(\pi^0 \rightarrow "e")$ is the probability that a $\pi^0$ event in the shower sample will make the final electron sample, and $P(\pi^0 \rightarrow "\pi^0")$ is the probability that a $\pi^0$ event in the shower sample will not make the final electron sample. There were 111 data events in the shower sample and 37 data events in the final electron sample. Therefore, there were 74 data events in the shower sample that do not make the final electron sample. However, the Monte Carlo predicts that 10.5 of these events are due to $\nu_e$ interactions, $\bar{\nu}_e$ interactions, or $\nu_e$ elastic scattering interactions. Therefore, $N_{\pi^0 \rightarrow "\pi^0"}$ is 63.5. $P(\pi^0 \rightarrow "e")$ and $P(\pi^0 \rightarrow "\pi^0")$ are calculated using $\nu_\mu$ event Monte Carlo to be 0.189 and 0.811 respectively. This leads to $N_{\pi^0 \rightarrow "e"}$ being 14.7 events.

The second method of calculating the number of $\pi^0$ events in the final electron
sample again using Equation 7.5 except now $P(\pi^0 \to "e")$ and $P(\pi^0 \to "\pi^0")$ are found using the data events in the M2 sample described in the data reduction chapter. The muon track in these events is eliminated in software and the secondary cluster is passed through the shower and final electron cuts. The secondary cluster cannot be an electron and is similar to the primary cluster in $\pi^0$ events in the shower and electron samples. 49 data M2 events with the muon eliminated pass the shower cuts and 9 pass the electron cuts. This gives $P(\pi^0 \to "e") = 9/49$ and $P(\pi^0 \to "\pi^0") = 40/49$. This method gives $N_{\pi^0 \to "e"} = 14.3$ events.

The two additional methods predict numbers of $\pi^0$ events in the final electron sample (14.7, 14.3) which agree well with the direct Monte Carlo prediction (13.7). Despite this agreement, the systematic error on the predicted number of events in the final electron sample due to $\nu_\mu$ interactions (other than coherent $\pi^0$ production and $\nu$-e elastic scattering), $\nu_\tau$ interactions, and coherent $\pi^0$ production is estimated at 30% because of the cross section uncertainties.

The Monte Carlo predicts that $36.2 \pm 4.7(sys) \pm 6.0(stat)$ events will pass all electron neutrino analysis cuts if neutrino oscillations do not occur. 37 data events do pass all electron neutrino analysis cuts.
Chapter 8

Conclusion

5,579 beam data events are in our final sample of quasi-elastic $\nu_\mu$ event candidates. The Monte Carlo predicts 5,592 events in this sample. The reconstructed neutrino energy spectra of these beam data events and the Monte Carlo prediction are shown in Figure 8.1. The good agreement confirms the Monte Carlo normalization and gives us confidence in our Monte Carlo.

Our analysis finds 111 beam data events with only a single electro-magnetic shower in the final state. The Monte Carlo predicts 94 of these events from sources other than neutrino oscillations: 33 events with an electron in the final state and 61 events with a misidentified $\pi^0$ in the final state. Our analysis finds 37 beam data events with only a single electron in the final state. The Monte Carlo predicts $36.2 \pm 4.7 (sys) \pm 6.0 (stat)$ events from sources other than neutrino oscillations: 22.5 events with an electron in the final state and 13.7 events with a misidentified $\pi^0$ in the final state. The reconstructed neutrino energy spectra for the data and Monte Carlo prediction at the two stages of analysis is shown in Figure 8.2.

No excess of $\nu_e$ events over those expected from sources other than neutrino oscillations was observed. No evidence for neutrino oscillations was found. 90% confidence
Figure 8.1: Comparison of the reconstructed $\nu_\mu$ spectra for the data (points) and the Monte Carlo prediction (histogram).
Figure 8.2: Reconstructed neutrino energy spectra for the shower sample. (a) Comparison of the data sample (with error bars) with the Monte Carlo prediction (solid histogram). (b) Contributions to the Monte Carlo prediction from $\pi^0$ events (solid histogram) and $e^-$ events (dotted histogram). Reconstructed neutrino energy spectra for the final electron sample. (a) Comparison of the data sample (with error bars) with the Monte Carlo prediction (solid histogram). (b) Contributions to the Monte Carlo prediction from $\pi^0$ events (solid histogram) and $e^-$ events (dotted histogram).
level limits are set on the oscillation parameters, $\sin^2 2\theta$ and $\Delta m^2$, using a maximum likelihood method as described in Appendix B. The limits we set are shown in Figure 8.3 with other recent results [45, 46, 47, 48, 49] for comparison.
Figure 8.3: 90% confidence level limits on oscillation parameters set by this and other recent experiments.
Appendix A

A2 Test Run

A miniature version of the E776 detector was placed in the A2 test beam at Brookhaven National Laboratory during the summer of 1986. The test detector was a miniature version of the E776 calorimeter and had no toroids. It consisted of 40 planes of 16 PDTs. As in the actual detector, the PDTs in consecutive planes were oriented orthogonally, one inch of concrete absorber was placed between each PDT plane and every tenth layer of concrete was replaced with scintillator. The PDTs were identical to those in the actual detector except only 54 inches in length. The concrete, scintillator, gas, and electronics were also identical to the actual detector except in dimensions perpendicular to the beam direction. The A2 test detector is shown in Figure A.1.

The beam line was instrumented to allow us to take data for different types of particles. There were two Čerenkov counters \((C1,C2)\) and two time of flight scintillators \((S1,S2)\) in the beam line. Immediately upstream of the test detector, there were four veto counters with a two inch opening for the beam. The trigger for electrons was

\[
S1W \cdot S2 \cdot (C1 + C2)
\]

where \(S1W\) is the \(S1\) signal delayed to put \(S1\) and \(S2\) in time for electrons. Cuts on
Figure A.1: The A2 test detector.
the veto counters were made later in software. The location of the beam line, test
detector, and beam instrumentation is shown in Figure A.2.

Test data was important in understanding the behavior of different particles in our
detector. The electron test data was used to determine the angular resolution, energy
resolution, and energy calibration for electrons in our full-sized detector. Test data
was taken for electrons ranging in energy from 300 MeV to 4 GeV with the detector
oriented at both 0° and 30° with respect to the beam.
Figure A.2: The A2 beam line.
Appendix B

Maximum Likelihood Method

We set our 90\% confidence level limits on the oscillation parameters using the extended maximum likelihood method [50].

First, we fix $\Delta m^2$ and then find the 90\% confidence level upper limit of $\sin^2 2\theta$ for that $\Delta m^2$. The likelihood of a particular $\sin^2 2\theta$ and overall normalization $\alpha$, $L(\sin^2 2\theta, \alpha)$, is

$$L(\sin^2 2\theta, \alpha) = e^{-N(\sin^2 2\theta, \alpha)} \prod_{i=1}^{N} F(\sin^2 2\theta, \alpha, E_i) \quad (B.1)$$

where the product is over all data events in the final electron sample.

$F(\sin^2 2\theta, \alpha, E)$ is simply the expected reconstructed neutrino energy spectrum if neutrinos do oscillate with the given $\Delta m^2$ and $\sin^2 2\theta$ with $\alpha$ as the overall normalization of the spectrum. The probability of getting an event in interval $dE$ is $F(E) dE$, and

$$\int_{cGeV}^{10GeV} F(E) dE = \bar{N}$$

is the number of events we expect to observe in the experiment. The probability of observing no events in some interval $\Delta E$ is

$$e^{-\int_{E}^{E+\Delta E} F(E) dE}.$$
The probability of observing no events at all is

\[ e^{-\int_{\text{background}} F[E]dE} = e^{-N} \]

The element of probability of finding N events at energies \( E_1, E_2, ..., E_N \) is

\[ d^N p = e^{-N} \prod_{i=1}^{N} F(E_i)dE_i \]

and

\[ L(\sin^2 2\theta, \alpha) = e^{-N(\sin^2 2\theta, \alpha)} \prod_{i=1}^{N} F(\sin^2 2\theta, \alpha, E_i). \]

In practice, the calculation is done by summing distributions with 400 MeV bins instead of integrating.

For our experiment,

\[ F(\sin^2 2\theta, \alpha, E_i) = \alpha(\text{BACK}(E_i)) \]

\[ + \nu_\mu \text{SPEC}(E_i) \times \nu_e \text{ACC}(E_i) \times \sin^2 2\theta \sin^2(\frac{1.27 \Delta m^2 L}{E}) \]

and

\[ N(\sin^2 2\theta, \alpha) = \sum_{i=1}^{N} F(\sin^2 2\theta, \alpha, E_i) \]

where the sum goes over all energy bins. \( \text{BACK}(E_i) \) is the binned reconstructed neutrino energy spectrum of events predicted in the final sample if there are no oscillations. \( \nu_\mu \text{SPEC}(E_i) \) is the binned true (not reconstructed) neutrino energy spectrum of interacting muon neutrinos expected in our detector. \( \nu_e \text{SPEC}(E_i) \) is the binned acceptance of \( \nu_e \) events as a function of the true neutrino energy.

\( \nu_\mu \text{SPEC}(E) \) and \( \nu_e \text{ACC}(E) \) are functions of the true neutrino energy because whether a \( \nu_\mu \) will oscillate is determined by its true energy not the energy we reconstruct it at. Our \( \nu_e \) event acceptance is also known as a function of true not
reconstructed neutrino energy. However, in effect, we have said we can perfectly reconstruct the neutrino energy of events due to oscillations. This is incorrect but an attempt to correct this by letting the neutrinos reconstruct imperfectly showed this had little or no effect on the final limits set on the oscillation parameters.

It should also be noted that \( \sin^2\left(\frac{1.27\Delta m^2 L}{E}\right) \) swings widely with small changes in \( E \) for large values of \( \Delta m^2 \). \( F(\sin^2 2\theta, \alpha, E_i) \) includes the contribution from oscillations and the energy of the potential oscillating neutrinos in a given bin varies by 400 MeV. \( L \) is also not single valued since the neutrinos are produced at different points in the decay tunnel. \( \sin^2\left(\frac{1.27\Delta m^2 L}{E}\right) \) is not single valued for the neutrinos in the bin. The value of \( \sin^2\left(\frac{1.27\Delta m^2 L}{E}\right) \) used is the average over the energies and lengths in the bin assuming the energy distribution is nearly flat over the width of the bin and the lengths are exponentially distributed.

To take into account systematic errors, the calculation of \( L(\sin^2 2\theta, \alpha) \) is actually a bit more complex than previously stated. The basic idea is that the probability of A and B occurring is the product of A occurring given B and the probability of B occurring. For simplicity, the systematic errors are assumed to be gaussian and to affect only the normalization not the shape of the energy spectrum of a given Monte Carlo prediction. The expected final reconstructed neutrino energy spectrum \( F(\sin^2 2\theta, E_i) \) now depends on two additional factors, \( f_1 \) and \( f_2 \), with

\[
F(\sin^2 2\theta, \alpha, E_i) = \alpha(f_1 \times EBACK_\nu(E_i) + f_2 \times EBACK_\nu^B(E_i))
+ \nu_\mu SPEC(E_i) \times \nu_\nu ACC(E_i) \times \sin^2 2\theta \sin^2\left(\frac{1.27\Delta m^2 L}{E}\right) \quad (B.4)
\]

where \( \nu_\mu SPEC(E_i) \) and \( \nu_\nu ACC(E_i) \) are as before. \( EBACK_\nu(E_i) \) is the binned reconstructed neutrino energy spectrum of events with an electron in the final state predicted in the final electron sample if oscillations do not occur. This prediction
has a systematic error of 10.5%. \( E B A C K_{\nu}(E_i) \) is the binned reconstructed neutrino energy spectrum of events with no electron in the final state predicted in the final electron sample if oscillations do not occur. This prediction has a systematic error of 30%. The probability of the normalization factors, \( f_1 \) and \( f_2 \) being the true factors, \( P(f_1, f_2) \), is a normalized two dimensional gaussian centered at \( f_1 = f_2 = 1 \) with \( \sigma_{f_1} = 0.105 \) and \( \sigma_{f_2} = 0.3 \).

\[ L(\sin^2 2\theta, \alpha, f_1, f_2) \] can be calculated from \( F(\sin^2 2\theta, \alpha, f_1, f_2, E_i) \). \( L(\sin^2 2\theta, \alpha) \) is then found from

\[ L(\sin^2 2\theta, \alpha) = \int_0^\infty \int_0^\infty P(f_1, f_2)L(\sin^2 2\theta, \alpha, f_1, f_2)df_1df_2. \]  

The 90% confidence level limit on \( \sin^2 2\theta, C \), is then found from

\[ \int_0^\infty \int_0^CL(\sin^2 2\theta, \alpha)d\sin^2 2\theta d\alpha = 0.9 \cdot \int_0^\infty \int_0^CL(\sin^2 2\theta, \alpha)d\sin^2 2\theta d\alpha. \]

The procedure is repeated for each value of \( \Delta m^2 \).
Bibliography


Vita

William Patrick Hogan was born in the Chicago area on November 6, 1963. He was raised in Oak Lawn, Illinois and graduated from Oak Lawn Community High School. He then began attending the University of Illinois in Urbana and graduated with a B.S. in physics in May, 1985. He continued at the University of Illinois receiving his M.S. in physics in August, 1986. He received his Ph.D. in physics in October, 1990 for a search for neutrino oscillations performed at Brookhaven National Lab with collaborators from Columbia University and the Johns Hopkins University. Throughout his life, Bill has voted for Democrats, attended mass on Sundays, and rooted for the White Sox and against the Cubs.